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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE

Mount Wilson Observatory of the Carnegie
Institution of Washington

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University of Chicago

JANUARY 1923

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JANUARY 1923

NUMBER 1

THE ELLIPSOIDAL VARIABLE STAR, *b* PERSEI

By JOEL STEBBINS

ABSTRACT

Variable star b Persei; photometric study.—During 1920–1922, 140 observations were made with a photo-electric photometer. The light-curve plotted from the normals, Figure 1, shows this star (magnitude 4.57; spectrum A₂) to be an ellipsoidal variable, with a period of 1.52732 days and a total range of variation of 0^m06, of which about 0^m05 is due to the ellipsoidal figure of the components of the binary system, and the rest may be ascribed to a periastron effect in the elliptical orbit. If there is any eclipse it is slight. The precision of the measurements is such that the probable error of a normal is $\pm 0^{\text{m}}0027$.

Comparison star λ Persei is probably constant in light.

The variability of *b* Persei was discovered at this observatory in 1920, and it was very soon found that the variation is due to the ellipsoidal figure of the components of this binary system, period 1.52732 days. Series of observations have been taken with the photo-electric photometer during the winters of 1920–1921 and 1921–1922.

Following are the Harvard data for the variable and the comparison star:

H.R. 1324, *b* Persei, magnitude 4.57, spectrum A₂

H.R. 1261, λ Persei, magnitude 4.33, spectrum A₀

It was not convenient to take another comparison star, and in the long series of observations there has been no evidence of variability of λ Persei.

In Table I are the observations. The first column gives the Julian date. The phase is computed from the final elements, the reduction of the times to the sun being neglected. Each difference of magnitude is derived from the number of sets of measures indicated, each set being the mean of four measures on each star taken symmetrically. The few measures which were rejected are placed in parentheses. Where haze or smoke is indicated, it was only suspected that the poor conditions may have affected the measures, though the readings themselves did not show it.

The observations in Table I were combined into the normal magnitudes in Table II in the usual way, each normal being the mean of some twenty to twenty-four sets of measures, and after once formed the normals were all considered to be of equal weight.

For a comparison with the light-curve we have the spectroscopic elements by Cannon.¹

$$\begin{aligned}P &= 1.52732 \text{ days} \\e &= 0.22 \pm 0.021 \\w &= 151^\circ 75 \pm 2^\circ 08 \\K &= 41.89 \text{ km/sec} \pm 0.97 \text{ km/sec} \\\gamma &= 23.09 \text{ km/sec} \pm 0.57 \text{ km/sec} \\T &= \text{J.D. } 2418956.166 \pm 0.02 \text{ day} \\a \sin i &= 837,600 \text{ km}\end{aligned}$$

The faint secondary spectrum was measured and gives

$$m_1 \sin^3 i = 0.85\odot, \text{ and } m_2 \sin^3 i = 0.23\odot.$$

From a graph of the normals there was derived the epoch of primary minimum which was found to occur 0.033 day later than the prediction from the spectroscopic elements. Cannon's period was derived from observations extending from 1903 to 1910, and the discordance between the light-curve and the prediction is accounted for by a correction of only +0.000013 day to his period. However, I have adopted the spectroscopic period unchanged for the present purpose, and the adopted light elements of *b* Persei are as follows:

$$\text{Minimum} = \text{J.D. } 2422780.433 + 1^d 52732 \cdot E$$

The phases in Tables I and II are computed from these elements.

¹ *Publications of the Dominion Observatory*, 1, 298, 1914.

TABLE I
OBSERVATIONS OF *b* PERSEI

Date, G.M.T.	Phase	$b - \lambda$	Sets	Remarks	Date, G.M.T.	Phase	$b - \lambda$	Sets	Remarks	
2422584.818...	1 ^d 409	o ^M 394	3		2422716.738...	o ^d 452 (o ^M 376)	6		Rejected	
2585.809...	o .873	.392	3		2718.594...	o .781	.392	6		
2586.781...	o .318	.350	3		.638...	o .825	.390	6		
2591.767...	o .722	.387	3		2723.600...	1.205	.355	5	Haze	
2592.760...	o .187	.369	3		2735.566...	o .953	.369	6		
2599.749...	1.067	.331	3		.592...	o .979	.367	6		
2601.731...	1.522	.417	3		.617...	1.004	.371	6		
2603.738...	o .474	.336	6		.640...	1.027	.356	6		
2605.703...	o .912	.386	6		.667...	1.054	.360	6		
2606.755...	o .437	.330	3		.731...	1.118	(-.354)	2	Test only	
2607.742...	1.423	.392	6		2737.592...	1.451	.396	6		
2609.685...	o .312	.335	3		.611...	1.470	.404	3		
2613.708...	1.280	.365	6		2738.566...	o .868	.368	6	Poor	
.766...	1.338	.376	6		.666...	o .938	.378	6	Poor	
2618.676...	o .139	.383	6		2742.629...	o .379	.340	6		
.753...	o .216	.361	3		2744.642...	o .865	.389	2		
2619.704...	1.167	.345	6		2751.631...	o .217	.383	3		
2627.683...	1.509	.398	6		2752.596...	1.182	.366	6		
2632.642...	o .359	.339	6		2758.572...	1.049	.379	6		
.723...	o .440	.316	3		.598...	1.075	.361	6		
2633.653...	1.370	.378	6		2759.576...	o .525	.349	2		
2639.724...	1.332	.386	3		2766.573...	1.413	.388	4		
2641.675...	o .228	.360	6		2778.572...	1.194	.344	6		
2642.662...	1.215	.350	6		2780.583...	o .150	.415	3		
2647.610...	o .063	.399	6		2954.703...	o .246	.368	6		
.704...	o .148	.307	6		.811...	o .264	.354	3		
2648.668...	1.112	.364	6		2955.815...	1.268	.360	6		
.738...	1.182	.361	3		2958.842...	1.240	.364	6		
2649.636...	o .553	.354	6		2967.799...	1.024	.364	6		
.703...	o .620	.381	3		2970.749...	o .928	.366	4		
2656.651...	I.458	.407	6		2975.728...	1.325	.365	3		
2669.590...	o .051	.383	6		2976.740...	o .810	.392	6		
.675...	o .736	.394	6		2977.706...	o .249	.358	6		
2670.684...	o .218	.357	3		.754...	o .297	.360	6		
2674.690...	1.169	.358	3		2983.786...	o .219	.382	6		
2675.580...	o .532	.358	6		2984.693...	1.126	.355	6		
.628...	o .580	.382	6		.746...	1.179	.348	6		
.649...	o .601	.363	3		2986.694...	o .073	.390	3		
2682.634...	1.477	.400	6		.740...	o .119	.392	5		
.672...	1.515	.404	6		3015.705...	o .065	.386	5		
.702...	o .017	.409	6		3033.707...	1.266	.363	6	Haze	
.750...	o .065	.406	6		3037.728...	o .705	.373	3		
.772...	o .087	.388	6		3043.655...	o .543	.359	6		
2683.704...	1.079	.359	6		3049.689...	o .448	.345	6		
.783...	1.008	.343	3		3054.704...	o .881	.388	6		
2692.690...	o .842	.374	3		.740...	o .917	.381	6		
2695.575...	o .672	.384	6		3056.754...	1.403	.392	6		
.608...	o .705	.388	6		3062.646...	1.286	.348	6		
.705...	o .802	.387	6		.719...	1.259	.363	3		
.754...	o .851	.392	6		3063.670...	o .683	.379	6		
.778...	o .875	.385	6		3068.669...	1.100	.362	6		
.795...	o .892	.398	3		.758...	1.180	.364	6		
2698.550...	o .592	.390	6		3069.783...	o .687	.390	6		
.644...	o .686	.395	6		3071.628...	1.004	.368	6		
.675...	o .717	.397	6		3076.666...	1.460	.413	6		
.706...	o .748	.368	6		.730...	1.524	.405	6		
.722...	o .764	.385	3		3087.614...	o .190	.384	6		
2699.642...	o .157	.385	6		3089.590...	o .638	.370	6		
.687...	o .202	.379	6		.620...	o .668	.393	3		
.742...	o .257	.354	6		.692...	o .740	.400	6		
2706.668...	1.104	.366	4		Poor	3101.643...	o .473	.340	6	
2716.528...	o .242	.363	6		3103.576...	o .878	.400	3		
.554...	o .268	.358	6		3105.642...	1.417	.398	2		
.584...	o .298	.358	6		3110.578...	o .135	.300	6		
.656...	o .370	.348	6		3117.575...	1.132	.359	6		
.678...	o .392	.352	6		3125.578...	1.498	.397	6		
.702...	o .416	.354	6							

Smoke
Smoke

Haze

Smoke

Smoke

The normals are shown in Figure 1, and it is evident that in addition to ellipsoidal variation there is a slight effect possibly

TABLE II
NORMAL MAGNITUDES

Phase	$b-\lambda$	Residual	Phase	$b-\lambda$	Residual
0.034	0.400	-0.003	0.867	0.391	+0.003
0.087	.396	.000	0.896	.383	.000
0.142	.395	+.009	0.935	.374	-.003
0.184	.381	+.004	1.003	.368	+.003
0.221	.369	+.001	1.047	.360	+.001
0.249	.361	-.001	1.087	.358	+.003
0.284	.358	+.003	1.120	.361	+.008
0.350	.342	-.004	1.175	.353	-.001
0.424	.344	-.002	1.189	.354	-.002
0.494	.345	-.008	1.227	.357	-.003
0.504	.371	+.005	1.279	.365	-.004
0.638	.378	-.002	1.365	.383	-.003
0.682	.388	+.001	1.427	.393	-.004
0.718	.390	-.001	1.466	.406	+.004
0.757	.397	+.003	1.509	0.402	-.002
0.817	0.387	-.005			

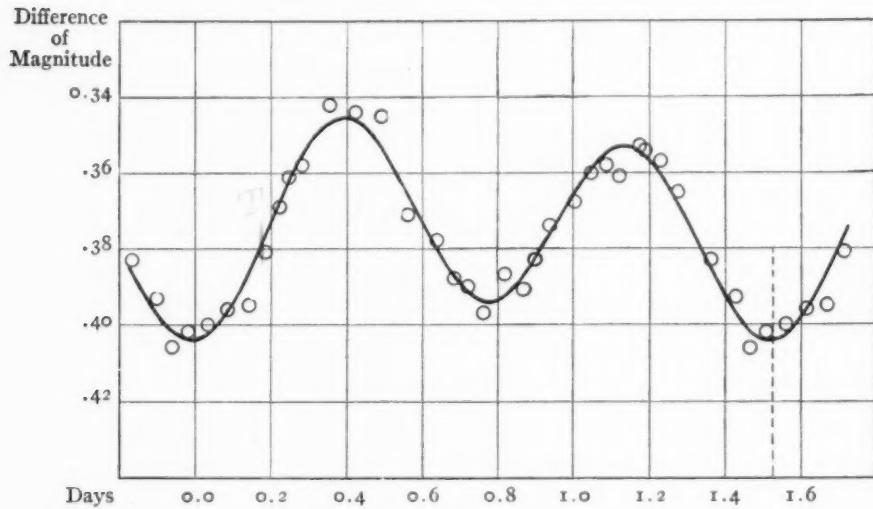


FIG. 1.—The light-curve of b Persei

due to the close approach of the bodies at periastron, the spectroscopic eccentricity of the orbit being 0.22. It seemed best to assume that there are no eclipses, and to represent the variation

with as few periodic terms as can be given a simple physical interpretation. From a least-squares solution of the equations set up from the thirty-one normals, I find the following:

$$m = +0^M 3492 - 0^M 0040 \sin \theta + 0^M 0052 \cos \theta + 0^M 0498 \cos^2 \theta \quad (1)$$

± 8	± 7	± 7	± 14
---------	---------	---------	----------

Here m is the magnitude referred to λ Persei, and the probable errors of the different terms are each given below the corresponding quantity. The terms in $\sin \theta$ and $\cos \theta$ take care of the combination of the periastron effect and a possible reflection of light from the adjacent sides of the two bodies, while the term in $\cos^2 \theta$ may be considered as due to the ellipsoidal figure.

The probable error of one normal magnitude is $\pm 0^M 0027$, and the probable errors of the different terms in (1) are all of the order of a thousandth of a magnitude. As far as accidental error is concerned, the results may be considered satisfactory, but there is always a possibility of systematic error in the correction for the atmospheric extinction, which, however, was usually less than $0^M 02$.

Equation (1) may also be written:

$$m = +0^M 3492 + 0^M 0066 \sin (\theta - 232^\circ 4) + 0^M 0498 \cos^2 \theta \quad (2)$$

± 8	± 7	$\pm 6^\circ 1$	± 14
---------	---------	-----------------	----------

The second term in (2) gives a maximum of light at phase 0.604 day or 0.429 day after periastron, but with the other complications there is no certainty that this difference is of any significance. There is the possibility that the apparent difference of about $0^M 01$ between the two minima is due to an eclipse at the predicted primary and not at secondary. The relative distances between centers of the bodies at these times are as 0.86 to 1.06 . It is sometimes difficult enough to separate the exact ellipsoidal variation from the eclipses in stars like β Lyrae, but in the case of b Persei there is no hope of separating any minute eclipse effect from the small continuous variation. One thing is obvious from the curve, however, and that is that the component bodies rotate with uniform motion, although the eccentricity of the orbit gives rise to large librations.

It may be noted that (1) or (2) gives the light minimum of the system at $\theta = -2^\circ 2$, or phase $-0^{\text{d}}009$, on the basis of the adopted elements. Also the single term in $\cos^2 \theta$ arbitrarily fixes the direction of the axes of the hypothetical rotating ellipsoids, whereas the use of terms in both $\sin 2\theta$ and $\cos 2\theta$ would leave this orientation to be determined by the solution. However, I have limited the number of terms to the fewest possible, and with the observations satisfied with a probable error of less than $\pm 0^{\text{M}}003$ for a normal magnitude, any further refinement seems superfluous. It might be that new spectroscopic measures would indicate a rotation of the line of apsides, and also verify the correction to the period.

A rough summary of the results for this system is that the light variation is due to the bodies being elongated at least 5 per cent, while there is possibly a slight periastron effect of about a hundredth of a magnitude.

I am indebted to Mr. C. C. Wylie for sharing the observations on this star. The work is part of that accomplished under grants from the Draper fund of the National Academy of Sciences.

UNIVERSITY OF ILLINOIS OBSERVATORY
June 1922

AN EXTENSION OF THE FUNDAMENTAL INFRA-RED ABSORPTION BAND OF HYDROGEN CHLORIDE

By W. F. COLBY, C. F. MEYER, AND D. W. BRONK

ABSTRACT

Fundamental absorption band of HCl at 3.4 μ .—With the help of a new grating of the echelle type, which has 2800 lines to the inch and throws most of the energy into the first order in the region 3.5 μ , this band has been extended from 3.9 to 4.2 μ . The seven new principal lines measured bring the total number to 39 and enable the constants of the empirical formula to be determined with considerable accuracy. Revised constants for Kratzer's theoretical formula are also given, and a table of all wave-numbers, observed and calculated. A new group of faint lines was found in the new region lying between the principal lines $m = -12$ to -17 , but with not quite the same spacing. The tests made prove the new lines to be real and not due to ghosts or reflections. They appear only when the gas is heated (500° C.), and they may correspond to a change of radial quantum from 1 to 2 although the frequencies do not agree with those predicted by Kratzer.

In a previous communication¹ two of us reported the results of measurements made upon the fundamental spectral absorption region of hydrogen chloride. The measurements were made for the most part on the side of the center toward short wave-lengths (positive side). This side of the band appeared at the time to be the most interesting, and we were under the impression that atmospheric absorption would set in upon the other side after the twelfth line.

Certain speculations of Kratzer² have lent interest to the investigation of the side of the band toward long wave-lengths (negative side), and the present work has to do mainly with measurements on this side. Upon actual examination it develops that atmospheric absorption does not set in until the eighteenth line, and even the nineteenth has been measured. The atmospheric absorption is due to carbon dioxide, and not to water-vapor as was erroneously stated.

We have used in this work a diffraction grating ruled by Dr. Barker of this laboratory upon the first Rowland ruling engine. The grating has a ruling of 2800 lines to the inch and throws most

¹ *Astrophysical Journal*, 53, 300, 1921.

² *Zeitschrift f. Physik*, 3, 289, 1920.

of the energy into the first order in the region 3.5μ . The amount of energy obtained from it is many times greater than with the 7500 line grating previously used, and it is possible to detect much smaller percentages of absorption. This additional power has led to the discovery of a series of faint lines between the principal lines in the region $m = -12$ to $m = -17$.

Figure 1 represents schematically the entire known hydrogen chloride spectrum, including what is new in the present investigation. The lengths of the vertical lines are roughly proportional to absorption percentage. In Figure 1b the principal lines of the fundamental are shown. Of these numbers -13 to -19 are now published for the first time. The faint lines in the fundamental are also shown. In Figure 1a the principal lines of the harmonic are shown, and the faint lines in the harmonic ascribed by Loomis and Kratzer to the isotope of chlorine.

The wave-numbers of the new lines of the fundamental band are given in Table I with the wave-numbers of the old lines subjoined to make the table complete. The wave-numbers are waves per decimeter. Table I thus includes 39 lines of the fundamental band, a sufficient number to make the determination of the constants in the empirical formula more reliable. The following formula has been determined by the method of least squares:

$$\nu = 28860.7 + 205.9831n - 3.010228n^2 - 0.02056583n^3.$$

Wave-numbers computed from this equation are given in Table I and also the differences between them and the observed values.

Kratzer has extended the general theory of band spectra in this region to cover, in particular, bands of the present type which are due to molecules in which the vibration is not harmonic. His formula for this band may be written with slightly different notation as follows:

$$\begin{aligned} \nu = & (W_0/h + h/8\pi^2I - a_1 - hu^2/8\pi^2I) + (h/4\pi^2I - 2a_1 - hu^2/2\pi^2I)m \\ & - (a_1 + 3hu^2/4\pi^2I)m^2 - (hu^2/8\pi^2I)m^3. \end{aligned}$$

In this formula m indicates the number of the line counting from the center. It is positive on the high frequency side and negative on the low frequency side. This usage is very convenient and has

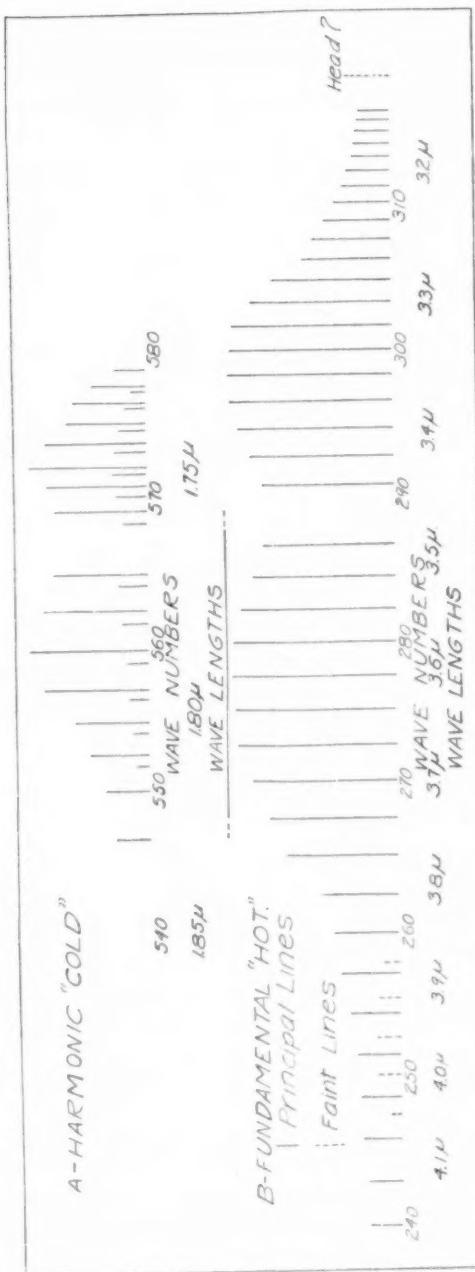


FIG. I

been customary in previous experimental work. It has the additional advantage of allowing one to represent both sides of the band by one equation. For the phenomenon of absorption, m thus indi-

TABLE I

n	ν (obs.)	ν (calc.)	Diff.
-19.....	24012	24001	11
-18.....	24299	24298	1
-17.....	24585	24590	-5
-16.....	24879	24879	0
-15.....	25163	25103	0
-14.....	25443	25443	0
-13.....	25717	25719	-2
-12.....	25987	25991	-4
-11.....	26252	26258	-6
-10.....	26520	26520	0
-9.....	26775	26778	-3
-8.....	27031	27031	0
-7.....	27278	27278	0
-6.....	27520	27521	-1
-5.....	27760	27758	2
-4.....	27991	27990	1
-3.....	28217	28216	1
-2.....	28437	28437	0
-1.....	28652	28652	0
1.....	29065	29064	1
2.....	29265	29260	5
3.....	29456	29451	5
4.....	29636	29635	1
5.....	29816	29813	3
6.....	29986	29984	2
7.....	30148	30148	0
8.....	30309	30305	4
9.....	30453	30456	-3
10.....	30597	30599	-2
11.....	30732	30735	-3
12.....	30858	30863	-5
13.....	30984	30985	-1
14.....	31095	31098	-3
15.....	31198	31204	-6
16.....	31298	31301	-3
17.....	31388	31391	-3
18.....	31481	31473	8
19.....	31554	31547	7
20.....	31615	31612	3

cates the original azimuthal quantum number and its sign indicates whether that quantum number has been increased or decreased during the absorption. Kratzer was limited in the application of the formula to hydrogen chloride to about half the number of lines

now available. The highest value of m which he could use was too low to determine the coefficient of the cubic term with any accuracy. This coefficient appears as a lower order term in the other coefficients and he has thus been able to neglect it. He did, however, determine the other three constants from the fundamental band and was able to evaluate the coefficient of the cubic term by relations between the fundamental and the first harmonic at 1.7μ . The values thus determined are

$$\begin{aligned}W'_0/h &= 28767.6 \\a_1 &= 2.975 \\h/4\pi^2 I &= 213.0 \\u &= 0.728 \text{ } 10^{-2}\end{aligned}$$

The present measurements seem to us to have sufficient precision to warrant considerable confidence in the empirical equation. If one calculates these four constants from our coefficients, the result is

$$\begin{aligned}W'_0/h &= 28757.7 \\a_1 &= 2.979 \\h/4\pi^2 I &= 211.96 \\u &= 0.6965 \text{ } 10^{-2}\end{aligned}$$

It must be remembered that these constants are computed from an entirely new set of experimental data which differs somewhat from that used by Kratzer. All the constants are therefore slightly altered. It is, however, very gratifying to find that the value of u determined from data in this band alone, agrees as closely as it does with the Kratzer value, which involves also the frequency of the harmonic.

The hypothesis of Kratzer, regarding the faint band, had as its only experimental basis at the time it was put forward, three faint absorption maxima shown in Imes's idealized curve, one between -9 and -10 , one between -10 and -11 , and the other between -11 and -12 . The present work was begun by attempting to establish or disestablish the existence of these faint maxima. While faint maxima were obtained in approximately the same positions in one trial, they have not been consistently found and consequently their existence has not been established, nor has it been definitely disestablished. To explain more clearly a few words may be in

order regarding the nature, and especially regarding the certainties and uncertainties of trials of the kind involved in this work. The spectrometer circle is set upon each minute of arc, and for the most part upon each half-minute of arc. At each setting, six or eight galvanometer deflections are taken, and these are averaged. When these averages are plotted against the circle settings the points may or may not fall upon a regular curve. If the points fall upon a regular curve, that is upon a curve of slight and more or less uniform curvature, it may be concluded that there is no absorption which is selective over the region investigated. But if the curve is irregular, and especially when it shows maxima, the question arises whether the irregularities are true, or due to accidental disturbances, chiefly of the galvanometer. If upon a second trial an irregularity appears in the same way, it is interesting; but only after it has appeared in four or five trials, always with the same outstanding characteristics, may it be supposed real, and then, while the irregularity may be safely supposed real, its cause is still uncertain. The supposed maxima between -10 and -11 , and -11 and -12 , which we may call $-X$ and $-XI$ respectively, if they exist at all appear only under some special conditions, the nature of which has not been ascertained, for while one trial showed irregularities, other trials give a smooth curve in the same place. Imes's experiments were carried out with a 15 cm absorption cell and the gas at room temperature. We have used the same absorption cell and also 60 cm cells both at room temperature and "hot," that is, heated to a faint glow which is visible in a darkened room without resting the eyes.

The faint lines of which the existence is established, namely those shown in Figure 1*b*, are obtained with the gas in the 60 cm cell "hot." It is necessary to heat the gas to bring out the absorption in the principal lines -13 to -19 , and when this absorption appears the faint lines are also found. They are not found when working with a 60 cm chamber at room temperature. Employing the same notation as before the faint lines may be called $-XII$, $-XIII$, $-XIV$, $-XV$, $-XVI$. Table II gives the wave-numbers of these lines. Figure 2 gives a curve of deflections inverted, that is, the deflections increase down the page. The principal lines -13 to

-16 show upon this curve of course, as well as the faint lines -XII to -XVI. The dotted line gives the energy-curve of deflections with no hydrogen chloride in the cell. The rise and fall in this curve is due to interference bands formed by the mica used as windows in the absorption cell, and more particularly before the thermopile. This same rise and fall appeared at regular intervals through the entire region explored whenever there was mica in the path, but disappeared when all mica was removed.

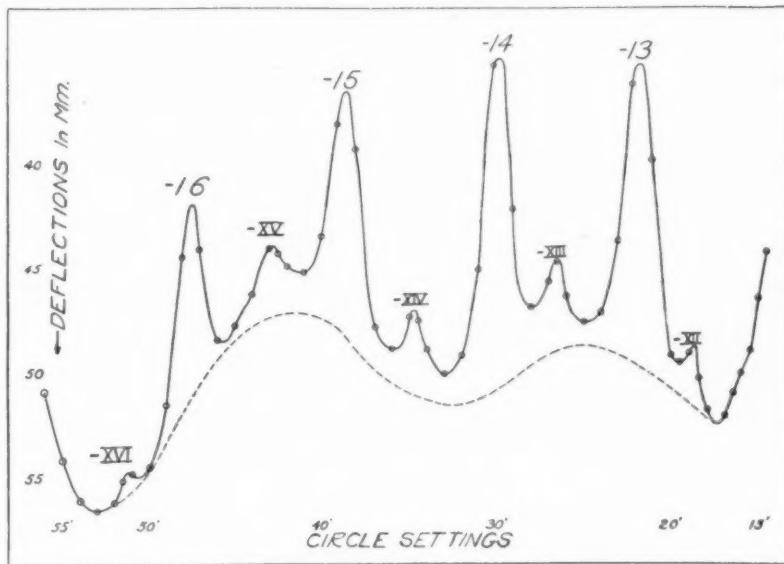


FIG. 2

Of the group -XII to -XVI the end lines are the faintest. The central one, -XIV, is probably the strongest, though it is difficult to make a definite gradation of the three lines -XIII, -XIV, and -XV. The absorption percentage for these three is about 7 per cent and for -XII and -XVI about 3 per cent, or in a galvanometer deflection of 45 mm, about 3 mm and 1.5 mm respectively. The absorption percentages of the principal lines -13 to -17 vary from 30 per cent to 12 per cent. It should further be noted that the position of the faint lines with reference to their neighbors is not uniform, but, instead, the faint lines have a characteristic spacing.

These faint lines do not fall into the band predicted by Kratzer, although their spacing does agree with the spacing which the lines of the predicted band would have in this region. This band was computed on the supposition that the absorption involved a change of the radial quantum number, n , from 1 to 2. Using Kratzer's constants the formula would read in our notation

$$\nu = 27901 + 201.30m - 2.895m^2 - 0.02259m^3.$$

Table II gives wave-numbers computed from this equation which lie in this region. It will be noticed that they differ from the observed lines by about 80 units. The first term of the equation involves the frequency of the harmonic, but a variation of this magnitude cannot be ascribed to errors in any of the experimental data used. We have, nevertheless, a group of lines which only appears when the gas is hot and which we must associate with a higher energy content than is possessed by the majority of the molecules at room temperature.

TABLE II

n	ν (obs.)	ν (calc.)	Diff.
-XII.....	25797	25871	-74
-XIII.....	25547	25621	-74
-XIV.....	25288	25367	-79
-XV.....	25021	25108	-87
-XVI.....	24760	24845	-85

We fortunately have at hand the original curves for this absorption band as observed by Imes. In his publication¹ he did not extend his curves to line -12 except in Figure 3, which he designates as an idealized curve. It is not a duplication of the actual observations and does not reproduce faithfully the irregularities between the lines "since they are not significant." Kratzer was forced to search for the faint lines of the series under these unfortunate circumstances and to assign wave-lengths to these faint maxima. This must have been very difficult indeed. If one turns to the original curves the difficulty is even greater. The variation of absorption percentage never rises to 5 per cent, which with Imes's small deflections could not have corresponded to a difference of galvanom-

¹ *Astrophysical Journal*, 50, 251, 1919.

eter reading of more than 0.5 mm. Such variations can be accepted as indications of real lines only under exceptionally favorable observing conditions and after many successful trials. Moreover the points are badly scattered and no smooth curve can be drawn through them. The present writers are of the opinion that Imes's data do not give any convincing evidence of faint lines in this region. If the present new group of faint lines be extrapolated over the region -12 to -9, it is found to lie very close to, and finally to coincide with, the principal lines. Even the line

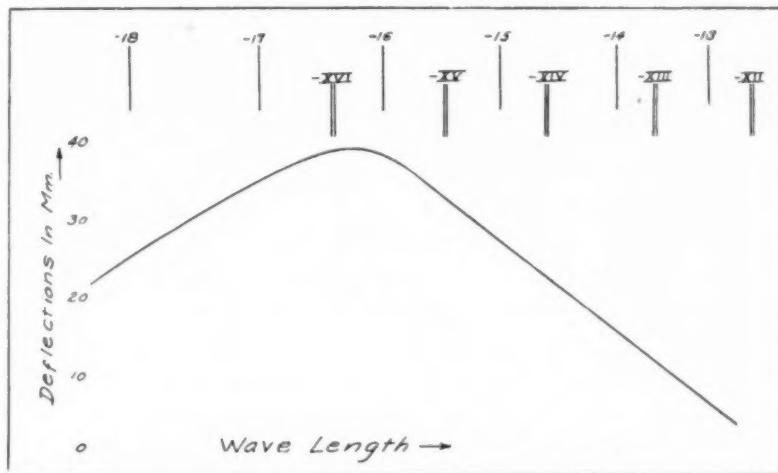


FIG. 3

near -12 lies so near as to require a resolution which our spectrometer barely furnishes.

In the use of a grating spectrometer there is under certain circumstances great danger of falling into error in the interpretation of the spectra obtained. This danger exists when faint lines are obtained in the same spectrum with strong ones, as in this investigation. The faint lines might be ghosts of the Rowland type due to periodic error in the grating, they might be Lyman ghosts, or they might be due to secondary reflections within the spectrometer. Reflections other than the principal ones intended in the design of the spectrometer sometimes occur, and may give rise to spurious lines if they happen to be in focus.

If the lines were Rowland ghosts one would expect to find them symmetrical on the two sides of each principal line, and this is not the case. The apparent wave-length of the ghosts of various orders can be calculated from the knowledge of the number of rulings of the grating per turn of the screw of the ruling engine. In this grating there are 144 rulings per turn so that a real line of wavelength 3.974μ would give rise to first order ghosts 4.0016μ and 3.9464μ , second order ghosts 4.0292μ and 3.9188μ , etc. In the region in which the faint lines have been found, this gives for the spacing of the first order ghost about 0.6 of the spacing of the principal lines. In the region of the center of the fundamental, the ghosts have about the same spacing as the principal lines, and so might easily coincide with them except at the very center of the band, where there is a missing line. Here the first order ghost from $+1$ and -1 should nearly coincide halfway between these lines, or the second ghost of $+2$ and -2 , etc. The principal lines are much stronger in this region than in the region -12 to -17 , yet a number of careful trials in the region between $+1$ and -1 yielded nothing except one slight irregularity near the base of -1 .

It seemed at first upon finding the faint lines that they might well be Lyman ghosts, let us say of the principal lines -8 to -13 , which are the only lines of the band which have a suitable spacing. But if they were, one would expect the intensities of the faint lines to decrease continuously in passing from $-XII$ to $-XVI$, since the intensities of the principal lines diminish in going from -8 to -13 . However, a more critical test of this question of Lyman ghosts seemed desirable. The entrance slit of the fore-prism spectrometer is as a rule open rather wide (e.g., 2 mm), in order to obtain more energy and a more nearly flat energy-curve. Then a comparatively wide spectral region falls into the grating spectrometer, wide enough to include more than the entire fundamental band, but still amply narrow to prevent overlapping orders of diffraction. To make the critical test, the entrance slit of the fore-prism spectrometer was narrowed to 0.35 mm, thus allowing energy to enter only as shown in the curve of Figure 3. The energy in the region of the strong lines of which Lyman ghosts were at first feared thus never entered the grating spectrometer. The faint lines were still present. In

order to restrict still farther the number of strong principal lines which entered the grating spectrometer, two trials were made with the fore-prism so set that the maximum of the energy-curve was on the side of long wave-lengths from the region examined, namely the region of $-XV$, so that the energy of the wave-length of line -14 had been cut down by 50 per cent and that of the line -13 by 90 per cent. The faint line still appeared in the same manner.

The same tests and arguments, which seem to preclude the possibility that the faint lines are Lyman ghosts, preclude also the possibility that they are secondary reflections.

It thus seems as though the conclusion were justified that the faint lines are a true part of the observed spectrum. Question still arises as to whether they are due to hydrogen chloride or due to an impurity. On account of their general similarity, that is, magnitude of spacing and regularity of both spacing and intensity, they may well be attributed to hydrogen chloride. There is no other known spectrum of which these lines might form a part. In the fundamental of hydrogen bromide lines -5 to $+2$ fall into the same region, but the lines do not agree with the faint lines here given, either in position or spacing. The fundamental of hydrogen iodide has not been found. It was carefully searched for by Mr. J. P. Cooley, of this laboratory, on both sides of the 4.2μ band of carbon dioxide, but not found. This means either that it is entirely covered by the carbon dioxide band, or else that hydrogen iodide is so highly dissociated even at room temperature that it does not yield its molecular spectrum.

As has been stated above, there may be other faint lines falling within the already explored region of the hydrogen chloride fundamental. The continuation of the new group may easily be covered by -12 to -9 . Proceeding from these in the direction of higher wave-numbers, the principal lines crowd more closely together and they overlap at the base, so that the chance of finding faint lines is greatly diminished. Proceeding from -17 toward lower wave-numbers, the absorption of carbon dioxide soon sets in, and at -19 is so strong that there is scarcely hope of obtaining observations beyond this, unless carbon dioxide were entirely excluded from the path of the beam.

The irregularity between -1 and $+1$, near the base of -1 , is interesting in connection with the speculation upon the possibility of existence of other faint lines. It was consistently found with a 15 cm cell with the gas at room temperature. It is probably not a ghost. On two trials with the 60 cm cell hot it failed to show up, but it may have been blotted out by continuous background (overlapping) and the galvanometer was not at its best. Extrapolation of the new group as far as this is very uncertain. Moreover, the line is very weak and is very near the strong line -1 . If we assign a provisional center to the new faint group at about 27810, it is possible to call this line $+4$. Higher temperatures might strengthen it and weaken the principal line -1 . Then it could be measured precisely and perhaps new faint lines would be revealed in this open region.

Some attempts have been made by Mr. H. L. Smith of the Ypsilanti Normal School, in collaboration with one of the writers, to photograph possible higher harmonics of the hydrogen chloride spectrum. This was tried both by absorption and emission. The region investigated was from 6500 Å to 9500 Å. The absorption in the higher harmonics would probably be faint, but it was thought that due to greater definition obtainable on the photographic film something might show. An absorption chamber about 12 cm in length was used and light from a Nernst glower passed through it. Filters were used to cut out overlapping orders. To minimize the effect of false illumination, as much of the spectrum on the side of short wave-lengths as possible was also cut out by filters, but no fore-prism spectrometer was used. In the experiments on emission, a spark was passed between monel metal electrodes in a flowing atmosphere of hydrogen chloride. Nothing was found attributable to the molecular spectrum.

The present contribution includes all lines of the 3.4μ band which are present with measurable intensity at this temperature, about 500° . Even with this grating we were not able to extend the band beyond $+20$ on the side of shorter wave-lengths, nor to find the head which probably lies not far beyond. The present chamber does not permit higher temperatures, but it is hoped soon to have a chamber of new design which will permit not only higher temperatures but

also a control of both temperature and pressure. This is of course essential to precise intensity measurements with which one may contribute experimental data to the question of the distribution of rotational velocities.

The authors wish to acknowledge their indebtedness to the American Association for the Advancement of Science for funds which assisted this investigation very materially.

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THE THERMAL IONIZATION OF GASEOUS ELEMENTS AT HIGH TEMPERATURES: A CONFIRMA- TION OF THE SAHA THEORY

BY ARTHUR A. NOYES AND H. A. WILSON

ABSTRACT

Saha's theory of thermal ionization.—The thermodynamic equation used by Saha for calculating the thermal ionization of the neutral atoms M of gaseous elements into positive ions M^+ and electrons E^- is first reviewed, especially with reference to the assumptions involved in the calculation. These assumptions are (1) that the ionization results from a chemical reaction $M = M^+ + E^-$ whose equilibrium or ionization constant K is given by the mass-action expression $K = p_M^+ p_{E^-} / p_M$; (2) that the increase in energy-content attending the ionization of each atom is equal to its ionization-potential V times the electronic charge e , and (3) that the heat-capacity and entropy of electron gas correspond to those of ordinary perfect monatomic gases. Introducing the best values of the constants involved there results the thermodynamic equation $\log_{10} K = -5048V/T + 2.5 \log_{10} T - 6.56$.

Calculation of ionization from the conductivities of salted flames.—The recent measurements of H. A. Wilson, as well as the earlier ones of Arrhenius, Smithells, Dawson, and Wilson, on the conductivity of flames into which salt solutions are sprayed, clearly indicate that the conductivity arises from an ionization process of the stated type; and Wilson's experiments make possible a computation of the relative magnitudes of the ionization-constants of the five elements, Li, Na, K, Rb, Cs. Absolute values of the ionization-constants are also derived from the flame conductances with the aid of previous rough determinations of the mobility of the electrons and of the number of them per cubic centimeter.

Confirmation of the Saha Theory.—The relative values of the ionization-constants of the five alkali elements computed from the flame conductances and those calculated from the ionization-potentials by the thermodynamic equation form two series of values which run closely parallel to each other; thus, though the constants vary 5700 fold, the greatest deviation between the two series is only about 40 per cent. Finally, although only the order of magnitude of the absolute values of the constants could be estimated, yet they are found to correspond well with the values calculated by the thermodynamic equation, these last being only from 1.1 to 2.3 fold larger.

I. INTRODUCTION

In a recent article Eggert¹ showed that the ordinary thermodynamic expression for the effect of temperature on the equilibrium of chemical reactions, when supplemented by assumptions as to the values of the special constants occurring in it, can be applied to the calculation of the extent to which under different conditions neutral atoms of the gaseous elements are converted into positive ions and free electrons. Saha² has shown that one of the most uncertain factors in

¹ *Physikalische Zeitschrift*, 20, 570, 1919.

² *Philosophical Magazine*, 40, 478, 809, 1920; 41, 267, 1921.

Eggert's formulation can be eliminated by introducing the ionization-potential of the element. He has then calculated by this method the dissociation of the first electron from the atoms of many important elements, thus, the extent to which such reactions as $\text{Na} = \text{Na}^+ + \text{E}^-$ take place, where E^- represents electron gas. He has tabulated the values of this percentage ionization at various temperatures and pressures, and has pointed out the great significance which these considerations may have for the interpretation of the spectra of elements under solar and stellar conditions.

These principles are of great interest to astronomers, as may be illustrated by the following applications. It has long been known that the solar prominences show strongly the hydrogen and helium lines and the enhanced (H and K) lines of calcium, but not the blue line of calcium nor the familiar lines of sodium or of other alkali elements. The remarkable absence of these lines had not been explained; but the thermodynamic relations now show that the extremely small pressure combined with the high temperature of the prominences must greatly promote the ionization of all elements, and may well suffice to convert the neutral calcium atoms into the calcium ions which produce the H and K lines; and also to convert the neutral sodium atoms, to which the D lines are due, completely into sodium ions, which do not yield strong lines in the visible spectrum. A second interesting application of Saha's equations has been made by H. N. Russell,¹ who was led to the discovery of rubidium in sun-spots by the considerations that the absence of its lines in the general solar spectrum is probably due to the complete conversion by the high temperature there prevailing of the neutral rubidium atoms (which produce the characteristic lines of the element) into rubidium ions (Rb^+), and that at the lower temperature prevailing in sun-spots this conversion might well be only a partial one.

2. THE THERMODYNAMIC EXPRESSIONS

The equilibrium-constant K of a reaction $M = M^+ + E^-$ at any definite temperature in terms of the partial pressures p_M , p_M^+ , p_E^- ,

¹ *Astrophysical Journal*, 55, 129, 1922.

of the three substances, regarded as perfect gases, is expressed by the following equations.

$$\frac{p_M^+ p_E^-}{p_M} = K. \quad (1)$$

$$\frac{x^2 p}{1-x} = K. \quad (2)$$

The second equation, in which x represents the fraction ionized and p the sum of the pressures p_M and p_M^+ , is valid only when $p_M^+ = p_E^-$; that is, only when electrons do not originate from any other source, as from the presence of another ionizing element or from thermionic causes.

The second law of thermodynamics leads to the following equations for the change of this equilibrium-constant with the temperature, in the case where the heat-content increase, ΔH , attending the reaction, can be expressed as a linear function of the absolute temperature, thus, by the formula $\Delta H = \Delta H_0 + T\Delta C_p$, where ΔC_p is the increase in the heat-capacity C_p at constant pressure that results from the occurrence of the reaction, namely:

$$d \ln K = \frac{\Delta H_0 + T\Delta C_p}{RT^2} dT. \quad (3)$$

$$\ln K = -\frac{\Delta H_0}{RT} + \frac{\Delta C_p}{R} \ln T + \frac{I}{R}. \quad (4)$$

In this equation I is the integration constant resulting from the integration of equation (3).

Now Saha introduces the following assumptions as to the quantities occurring in this equation:

- (1) That the energy increase attending the ionization of a single molecule is equal to the electrical work Ve that must be expended in order to give to an electron a kinetic energy which just suffices to ionize an atom of the element with which it collides; e being the charge on the electron and V the ionization-potential, which last may be determined by direct measurement or computed from the spectral series of the element.
- (2) That the heat-capacities of the neutral atom M and its ion M^+ are equal, and that the heat-capacity of electron gas is the

same as that of any other monatomic gas, namely, $2.5 R$ per mol of the gas.

(3) That the constant I , which is the sum, $I_M^+ + I_E^- - I_M$, of constants¹ characteristic of the three separate substances, can be evaluated by assuming $I_M^+ = I_M$, and calculating I_E^- , the constant for electron gas, from a relation apparently applicable to ordinary monatomic gases in the way indicated in the following paragraphs.

Namely, it follows from the general principles relating to entropy, and from the fact that the molal heat-capacity at constant pressure of perfect monatomic gases at all temperatures is $\frac{5}{2}R$, that the entropy S of one mol of any such gas at any temperature and pressure is given by the following equation, in which S_i is a constant characteristic of the gas:

$$S = S_i + \frac{5}{2}R \ln T - R \ln p.$$

Now the considerations of Sackur,² Tetrode,³ Stern,⁴ Tolman,⁵ and Lewis⁶ have led to the conclusions that the quantity S_i is a function only of the molecular weight M of the gas, and that the constants occurring in the functional relation can be evaluated with fairly concordant results by certain methods that need not be here described. Adopting the value obtained by Lewis from the theory of ultimate rational units,⁷ and shown by him to be concordant within $\frac{1}{2}$ to 2 per cent with the experimentally determined entropies of the four best studied monatomic gases (helium, argon, cadmium, and mercury)⁸ we get, in calories per degree and for the pressure in atmospheres

$$S_i = -2.63 + R \ln M. \quad (5)$$

¹ These constants are $2.303 R$ times greater than the "chemical constants" employed by Nernst (*Theoretische Chemie*, 7th ed., p. 742).

² *Annalen der Physik*, **36**, 598, 1911; **40**, 67, 1913.

³ *Ibid.*, **38**, 434; **39**, 255, 1912.

⁴ *Zeitschrift für Electrochemie*, **25**, 66-80, 1919.

⁵ *Journal of the American Chemical Society*, **42**, 1185, 1920; **43**, 1593, 1921.

⁶ *Physical Review*, **18**, 121, 1921.

⁷ Lewis and Adams, *Physical Review*, **3**, 92, 1914.

⁸ Lewis, Gibson, and Latimer, *Journal of the American Chemical Society*, **44**, 1009, 1922.

Substituting for M the value 5.4×10^{-4} , we find for electron gas $S_1 = -25.02$. Finally, it can be shown thermodynamically that the integration-constant I above considered is less than the entropy constant S_1 for a monatomic gas by its molal heat-capacity C ; that is, $I = S_1 - C = S_1 - 4.96$. There is thus obtained for I/R the value -15.10 .

These considerations lead then to the following values of the constants occurring in equation (4):

$$\frac{\Delta H_0}{R} = \frac{NeV}{R} = \frac{96500 V}{8.316} = 11625 V; \quad \frac{\Delta C}{R} = 2.5; \quad \frac{I}{R} = -15.10.$$

Substituting these values in equation (4) and changing from natural to ordinary logarithms, we get finally:

$$\log K = -\frac{5048 V}{T} + 2.5 \log T - 6.56. \quad (6)$$

3. THE ELECTRICAL CONDUCTIVITY OF FLAMES AND ITS INTERPRETATION

It will now be shown that the investigations of Arrhenius, Smithells, Dawson, and H. A. Wilson on the conductivity of flames containing salts have led to results which clearly indicate that the conductivity arises from ionization reactions of the type considered in the previous section of this article.

The final experiments of Wilson¹ were made by spraying solutions of various salts of known concentrations into a mixture of air and gasoline, which then passed through a burner consisting of a series of parallel quartz tubes, whereby a large flame was produced. Near the sides of this flame were introduced two vertical strips of platinum, by means of which a steady current was passed through the flame. Between these electrodes were inserted in the heart of the flame two horizontal platinum wires; and the potential-difference between these two wires (as measured with a quadrant electrometer) for a given current was taken to be proportional to the resistance of the flame between them. By employing the potential-difference at these wire electrodes, instead of that between

¹ *Philosophical Transactions of the Royal Society, A*, 216, 63-90, 1915.

the plate electrodes where the current entered the flame, the effect of the large potential-gradient around these latter electrodes was eliminated. Two such flames were always operated in series with the same current passing through them; and their conductances were compared by measuring the potential-differences between the wires in the two flames. By varying the concentration of the solution sprayed into one of the flames and keeping the other concentration unchanged, the variation of the conductance with the concentration and with the nature of the salt was obtained. The temperature in the heart of the flame, as measured with a thermocouple, registered about 1650° C. Since the couple is considerably cooler than the flame itself, we will assume the temperature to be 2000° K.

The general conclusions originally drawn from his measurements by Wilson, so far as they have a bearing on the present considerations, are summarized in the following paragraphs.

1. The conductance is independent of the acidic constituent of the salt, as was first shown by Arrhenius¹ in 1891. Thus the conductance was found by Wilson² to be identical when equivalent quantities of sodium chloride and carbonate, or of potassium chloride and carbonate, were sprayed into the flame. These facts show that the basic and acidic constituents of the salts must be completely separated in the flame; thus that both NaCl and Na₂CO₃ are completely dissociated, with the help of the water present, into Na₂O and HCl or Na₂O and CO₂, or into Na, HCl, and O₂, or Na, CO₂, and O₂. The latter conclusion is the one to which the considerations presented below lead. Chemical equilibrium data are not available for determining whether the oxides of the alkali elements are dissociated into their elements at the temperature and partial pressure prevailing in the flame; but this conclusion is made a priori probable by the facts that the partial pressure of the salt is extremely small (of the order of 10^{-8} atmospheres), and that in the heart of the flame, where the combustion is still incomplete, reducing gases such as carbon monoxide and hydrogen are probably present and the partial pressure of free oxygen is relatively small.

¹ *Annalen der Physik*, 43, 18, 1891.

² *Op. cit.*, pp. 76, 81.

2. The conductance of a flame containing a salt of any alkali element was shown by Arrhenius to be roughly proportional to the square root of the concentration of the salt solution sprayed into it. As seen from equation (2) above, this is to be expected if the ions are produced by a unibimolecular reaction (one by which two molecules are produced from a single one), and if the fraction of the salt ionized is small.

Wilson¹ found that the relation between the conductance L and the concentration c can be expressed over the whole range of concentration by the equation:

$$10^4 c = \frac{L^2 - 1}{L} (b + a L). \quad (7)$$

In this equation b and a are constants, the values of which were determined for a number of alkali salts. When the conductance is large, this equation becomes approximately $10^4 c = a L^2$, so that the conductance then varies as the square root of the concentration, as found by Arrhenius.

Equation (7) was obtained in the following way, under the assumptions that both the salt and some constituent of the flame are ionized with formation of electrons and positive ions, and that equilibrium prevails between these and the unionized parts of the two ionizing substances.

Let p_E denote the partial pressure due to the electrons in the flame, p_M^+ that due to the positive ions of the salt, p_M that due to the neutral atoms of the salt, p_F^+ that due to the positive ions of the ionizing flame-substance F , and p_F that due to the unionized atoms of this substance; also let K be the ionization-constant of the element of the salt sprayed into the flame, and K_F be that of the ionizing flame-substance. The expressions for the mass-action are then as follows:

$$K = \frac{p_M^+ p_E}{p_M} \quad (8)$$

$$K_F = \frac{p_F^+ p_E}{p_F} \quad (9)$$

¹ *Op. cit.*, pp. 78-80

Since the conductance L is proportional to the number of electrons present, the conductance of the positive ions being negligible because their mobilities are small compared with that of the electrons, we have

$$AL = p_E = p_M^+ + p_F^+. \quad (10)$$

where A is a constant for all substances, equal to the ratio of the partial pressure of the electrons to the conductance of the flame.

Furthermore, the concentration c of the solutions of the various salts sprayed into the flame (when expressed in equivalents per liter) is proportional to the sum of the partial pressures of the neutral atoms and positive ions, since the conditions of spraying were always the same. Hence, denoting by B a constant that has the same value for all the substances, we have

$$Bc = p_M + p_M^+. \quad (11)$$

Combining equations (8), (9), (10), and (11), we get

$$AL = \frac{KBc}{AL+K} + \frac{K_F p_F}{AL}.$$

Placing, as Wilson did in recording his results, the conductance of the salt-free flame equal to unity, that is, placing $L=1$ when $c=0$, we get $K_F p_F = A^2$, and therefore, assuming p_F^+ small compared with p_F so that p_F is a constant

$$c = \left(\frac{L^2 - 1}{L} \right) \left(\frac{A}{B} + \frac{A^2 L}{KB} \right). \quad (12)$$

Comparing this equation with equation (7) used by Wilson, we see that they become identical if we place

$$\frac{A}{B} = \frac{b}{10^4}, \quad \text{and} \quad \frac{A^2}{KB} = \frac{a}{10^4}.$$

From these equations there follows for the ionization-constant K the relation

$$K = A \frac{b}{a} = \frac{p_E b}{La}. \quad (13)$$

As an example of the satisfactory way in which equations (7) and (12) represent the variation of the conductance with the concentration, the observed conductances of cesium chloride¹ are shown in Table I, beside those obtained by plotting the curve corresponding to equation (7) when the constants b and a are taken equal to 10 and 1, respectively, and c is in grams per liter.

Further evidence that the reaction which gives rise to the flame conductivity is essentially a unibimolecular one is afforded by the fact that the conductance of flames fed with mixtures of two different alkali metals is found to be that predicted by the mass-action law. This law requires that the ionization of each element, and especially that of the less ionized one, be reduced by the elec-

TABLE I
VALIDITY OF THE MASS-ACTION RELATION

Grams per Liter	Conductance Observed	Conductance Calculated	Grams per Liter	Conductance Observed	Conductance Calculated
0.00.....	1.00	1.0	0.08.....	22.7	23.8
0.0032.....	2.88	2.9	0.16.....	32.8	35.4
0.008.....	5.72	5.5	0.8.....	85.2	84.5
0.016.....	8.9	8.7	8.0.....	282.	278.
0.032.....	13.5	13.7	80.0.....	883.	890.

trons arising from the other; and this was shown to be the case by Arrhenius, H. A. Wilson,² and A. B. Bryan.³ Thus, without entering into the details of the calculation, it may be mentioned that a solution 0.171 normal in Na_2CO_3 and 0.00238 normal in K_2CO_3 (which salts when used separately gave conductances of 21.3 and 23.3, respectively, the sum being 44.6), produced a conductance of 32.2, while the conductance calculated was 31.55.

3. The conductivity of the flame arises from the presence of positive ions and electrons; but the mobility of the positive ions is so much smaller that their conductance can be neglected in comparison with that of the electrons. Thus Wilson⁴ found the mobility of the positive ions produced from the various alkali salts to be about 1 cm per second per volt per cm, while he had previously

¹ *Op. cit.*, p. 78.

³ *Physical Review*, 18, 285, 1891.

² *Op. cit.*, p. 87.

⁴ *Op. cit.*, p. 71.

found,¹ by measuring the Hall effect, the mobility of the negative ions to be 2450 cm per second. These last measurements were made with flames containing different alkali salts, and the mobility was found to be very nearly the same for all salts, as it should be if the negative ions are always electrons. The fact that the negative ions have a much greater velocity than the positive ions can be explained only by supposing the negative ions to be electrons, and the positive ions to be atoms or molecules.

These various facts make it fairly certain that the conductance arises from a unibimolecular reaction by which a positive ion and an electron are produced directly from a substance whose partial pressure is proportional to the concentration of the unionized part of the salt sprayed into the flame. As there seems to be no substance other than the neutral element which could ionize in accordance with these conditions, we may conclude that the flame conductance arises from complete conversion of the chloride or carbonate into HCl or CO₂, oxygen, and the alkali element, and partial ionization of the latter in accordance with equations such as Na=Na⁺+E⁻, where E⁻ represents the electron.

4. CORRESPONDENCE BETWEEN THE IONIZATION VALUES DERIVED FROM THE FLAME CONDUCTIVITIES AND FROM THE THERMODYNAMIC EQUATION

The thermodynamic equation (equation 6) applied to two different elements, whose equilibrium-constants are K₁ and K₂ and ionization-potentials V₁ and V₂, leads to the following expression:

$$\log_{10} \frac{K_2}{K_1} = \frac{5048(V_1 - V_2)}{T}. \quad (14)$$

Now the values of the ratio b/a of the constants of Wilson's equation are shown by equation (13) to be proportional to the ionization-constants K of the respective elements. Hence, if we assume the value of the ionization-potential for one alkali element, we can calculate it for the others by equation (14) from Wilson's values² of b/a. These are given in the second column of Table II. Taking the ionization-potential of sodium to be 5.111 volts and the tempera-

¹ *Physical Review*, 3, 375, 1914. ² *Op. cit.*, p. 82.

ture of the flame to be 2000° K. the other ionization-potentials are calculated to have the values given in the third column of the table. Beside them, in the fourth column under the heading "Observed," are placed the values tabulated by Hughes,¹ as calculated from the frequencies of the spectral lines according to the quantum theory, which values agree closely for the most part with those derived from the direct measurements by Mohler, Foote, and their collaborators.

TABLE II
OBSERVED IONIZATION-POTENTIALS COMPARED WITH THOSE CALCULATED
FROM FLAME CONDUCTANCES BY THE THERMODYNAMIC EQUATION

	VALUES OF b/a	IONIZATION-POTENTIAL V	
		Calculated	Observed
Cesium.....	10.00	4.00	3.873
Rubidium.....	2.25	4.26	4.154
Potassium.....	1.38	4.35	4.317
Sodium.....	0.0160	5.111
Lithium*	0.0023	5.46	5.362

* The value of b/a for lithium was obtained by using the results on the conductivity of flames containing lithium salts given by Smithells, Dawson, and Wilson (*Phil. Trans. Royal Soc., A*, **193**, 108, 1899), which show that lithium salts give nearly the same conductance as sodium salts when the concentration of the lithium is about seven times that of the sodium. Hence the value of b/a for lithium is one-seventh as great as that for sodium, since the constant a is proportional to the (large) concentrations which give equal conductances.

It will be seen that the calculated and observed ionization-potentials not only change in the same order, but that the values are of the same general magnitude. Especially noteworthy is the very large change in both series in passing from sodium to potassium, and the much smaller differences between the other elements.

Instead of calculating the values of the ionization-potential by assuming its value for sodium, we may calculate relative values of all the ionization-constants by equation (14) by using the known values of the ionization-potential. The values of K so obtained should be proportional to the values of the constant b/a obtained from the flame conductivities, or the product Ka/b should be constant for the various elements. Placing the value of K for lithium equal to unity we get the results shown in Table III. The values of the product Ka/b are seen to vary to the extent of about ± 40 per cent from the mean value.

¹ *Bulletin of the National Research Council*, **2**, 168, 1921.

These results confirm the validity of the term containing the ionization-potential in the Saha equation; but they do not confirm the heat-capacity and integration-constant terms, since these are

TABLE III
COMPARISON OF IONIZATION-CONSTANTS DERIVED FROM THE IONIZATION-POTENTIALS BY THE THERMODYNAMIC EQUATION WITH THOSE DERIVED FROM THE FLAME CONDUCTANCES

	<i>V</i>	<i>b/a</i> from <i>L</i>	<i>K</i> from <i>V</i>	<i>Ka/b</i>
Cesium.....	3.873	10.00	5731.	573
Rubidium.....	4.154	2.25	1119.	497
Potassium.....	4.317	1.38	434.	315
Sodium.....	5.111	0.0160	4.3	269
Lithium.....	5.362	0.0023	1.0	435
Mean.....	418

eliminated in the calculations. It will now be shown that the conductivity data lead also to absolute values of the ionization-constants which are at least of the same order of magnitude as those calculated by the thermodynamic equation.

Equation (13) shows that the ionization-constant *K* of an element is equal to $p_E b / L a$ in terms of the conductances *L* and the constants *a* and *b* derived from them. For a flame free from salt *L* was taken equal to unity, so that $K = p'_E b / a$, where p'_E denotes the pressure of the electrons in a flame free from salt. Now it was found¹ that a flame into which no salt was sprayed, such as was used in the above-described experiments, has a specific conductance of 5×10^{-7} reciprocal ohms, and that the mobility of the electrons, as estimated from the Hall effect, is 2450 cm per second for one volt per centimeter. The specific conductance \bar{L} is, however, substantially equal (since the positive ions have only negligible conductance) to the product of the electron mobility U_E times the charge F on one equivalent of electrons times their concentration c_E (in mols per cubic centimeter); that is,

$$\bar{L} = U_E F c_E ; \quad \text{or} \quad \bar{L} = \frac{U_E F p'_E}{RT}, \quad \text{since} \quad p'_E = c E R T .$$

whence

$$p'_E = \frac{R T \bar{L}}{U_E F} = \frac{82.07 \times 2000 \times 5 \times 10^{-7}}{2450 \times 96500} = 3.5 \times 10^{-10} \text{ atm.}$$

¹ Wilson, *Philosophical Magazine* (6), 10, 475, 1905.

An independent estimate of the partial pressure of the electrons in flames not fed with salt is afforded by the work of Wilson and Gold,¹ who from the apparent specific inductive capacity for high-frequency currents found the number n_E of electrons per cubic centimeter to be 1.1×10^9 . Their partial pressure at 2000° K. is therefore given by the following equation, in which N denotes the number of molecules in one mol:

$$p'_E = \frac{n_E}{N} RT = \frac{1.1 \times 10^9 \times 82.07 \times 2000}{6.06 \times 10^{23}} = 3.0 \times 10^{-10} \text{ atm.}$$

Using the first of these values of this partial pressure the ionization-constants given in the following table were obtained by the equation $K = p'_E b/a$, derived from equation (13) by placing $L=1$. The values of K derived from the ionization-potentials by the thermodynamic equation (equation 6) for a temperature of 2000° K. are also given for comparison.

TABLE IV
VALUES AT 2000° ABSOLUTE OF THE IONIZATION-CONSTANT K
MULTIPLIED BY 10^{12}

	Cs	Rb	K	Na	Li
From flame conductances	3500	780	430	5.6	0.8
From the thermodynamic equation . . .	8200	1600	620	6.2	1.4

It will be seen that these two sets of values do not differ by a factor of more than two or three. Since the value of the partial pressure of the electrons was derived from measurements of a flame different from that used in the conductivity measurements, the agreement is as good as could be expected. The results, therefore, afford a striking confirmation of the general validity of the calculation of the ionization of the elements from their ionization-potentials by the thermodynamic equation employed by Saha.

PASADENA, CALIFORNIA, AND HOUSTON, TEXAS
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¹ *Philosophical Magazine* (6), 11, 484, 1906; also Wilson's *Electrical Properties of Flames*, p. 110.

FILM DISTORTION AND ACCURACY OF PHOTOGRAPHIC REGISTRATION OF POSITION¹

By F. E. ROSS

ABSTRACT

Effect of photographic film distortion on the distance apart of point images.—The results of the investigations by Schlesinger, Perrine, and Albrecht are summarized. In this investigation photographs of artificial stars were made with a precision camera reducing in the ratio of 1 to 20, on plates one inch wide pressed firmly against a rigid metal frame. In agreement with previous results, no evidence of systematic errors in distance was found, indicating that, in general, distortions are of purely local origin and extent. While the average measurements of ten exposures on a triple-coated plate are practically the same as for ten exposures on a thin emulsion, the local distortions are greater on the thick film. Yet the greatest variation from the mean was only 0.0044 mm. Faint and normal exposures gave the same average measurements within 0.0003 mm. Examination of the images showed that the effects of local variations in distribution of the silver-bromide grains and of variations in sensitivity and in development are negligible. Intensification modifies the minor details of image structure, but does not remove the larger irregularities in the original images nor change the center of gravity appreciably. Measurements of the same plates, when wet and dry, agree accurately and indicate that on drying the images move perpendicular to the surface. However, this is not the case for images within a few millimeters of the edge, as these move outward on drying sometimes as much as $1/20$ mm. This effect may be reduced by drying with alcohol or by hardening with formalin. It is concluded that the principal factor in producing the slight image displacements observed is local non-homogeneity of the gelatine.

Accuracy of measurements of the position of stellar images.—The probable error is found to be considerably less in the case of plates with a fine-grain thin emulsion (astronomical) than for those with thicker emulsions (Seed 30 and triple-coated). Intensification of weak images also decreases the error. In the foregoing measurements the probable error of a single setting was about 0.0005 mm, whereas displacement due to film distortion varied up to 0.002 mm as a maximum, but averaged only 0.0005 mm for normal exposures on the thin emulsion.

The subject of film distortion, by which is meant relative displacement of photographic images due to any one of a number of possible causes, has occupied the attention of many investigators during the past twenty years. The present paper will be confined to the particular case of images relatively far apart or beyond the sphere of mutual influence.² It will be of interest to outline the investigations and conclusions of a number of workers in this field. Only the most recent and important investigations will be reviewed.

¹ Communication No. 154 from the Research Laboratory of the Eastman Kodak Company.

² The distortion affecting images which are very close together has been treated by the writer in previous papers. See *Astrophysical Journal*, 52, 1920; 53, 1921.

There appears to have been but very little duplication of effort, each investigator treating the subject from an original and attractive viewpoint, so that the totality of results forms a well-rounded whole. The vicissitudes to which a photographic plate is exposed in the various operations of developing, fixing, and drying, combined with the very obviously discrete structure of the photographic image, a structure subject to the vagaries of grain clumping, leave a great deal to the imagination of the investigator in a serious questioning of the faithfulness of the photographic impression.

The distortion of photographic film has been investigated by Dr. F. Schlesinger,¹ in 1906, in a manner which avoids several objectionable features of some previous investigations. Two methods were used. The distinctive feature of the first method consisted in a double development of the plate which had been exposed to a star field. The plate being measured after each development by two observers, the distortions, if present, of either an accidental or a systematic kind, can be differentiated by a simple mathematical process from the errors of bisection. No systematic distortion was found. The values of the distortion of an accidental nature and the accidental errors of measurement were as follows:

Distortion (mean error).....	.0009 mm
Error of bisection of an image (mean error) ..	.0020 mm

from which it is concluded that the plate distortions were small and less than the errors of measurement. Nine subsequent prolonged soakings in water of the same plate failed to increase this very minute distortion. Concerning the conclusions to be derived from such a test, the author remarks: "In the preceding experiments the first measurements were made from plates that had already gone through the processes of development. There is a possibility that the film becomes set in some way in the drying after development so that thereafter it has a tenacity that it did not have before." To overcome this objection, a plate was spattered with ink, giving fine sharply outlined dots. The co-ordinates of the dots were measured, after which the plate was immersed in a developing solution in

¹ "On the Distortion of Photographic Films," *Publications of the Allegheny Observatory* 1, No. 1.

which the developing agent itself was left out, then fixed, washed, and dried in the usual manner and remeasured. The mean result from five plates was

Mean distortion error0011 mm
Mean bisection error0017 mm

agreeing with the result of the first experiment. The author concludes that distortion errors are not to be feared.

Important additions to the literature are contributed by C. D. Perrine.¹ Perrine remarks: "It has been the experience of practically all astronomers engaged in the measurement and reduction of photographs of stars or of spectra that the discordances found among such measures are much larger than can be accounted for by the errors of measurement alone." Perrine investigated the discordance or distortion on three kinds of plates, Seed 23, 27, and Seed Transparency, all coated on plate glass. Plates were exposed in the laboratory to a pattern plate containing small holes as a check upon stellar exposures. Distances of the order of 5 mm were measured. His results are summarized in Table I. By "range" is

TABLE I
MEAN RANGES

	Laboratory Exposures	Stellar Exposures
Seed 23.....	.0051	.0051
Seed 27.....	.0051	.0043
Seed Transparency.....	.0021	.0026

meant the difference between the greatest and least result of measurement, or the dispersion in readings. Perrine finds that the error of bisection forms but an insignificant part of the foregoing, the range of the bisection error being .0006 mm (mean of five settings, direct and reversed). The agreement between laboratory and telescope exposures is considered to place the burden of the difficulty on the photographic plate. Accepting the result of numerous investigations, that distortion of the film itself is negligible,

¹ "Some Results of a Study of the Grains and Structure of Photographic Film," *Lick Observatory Bulletin*, No. 143, 1908.

the author concludes that the difficulty is to be found in the images themselves or more specifically in the "structure of the image." Examination under high magnification of a number of images, given identical exposures, discloses such diversity of outline that no doubt remains in the author's mind but that a true cause has been found. He concludes:

Broadly speaking, we may say that the trouble is due to a lack of homogeneity in the structure of the film. If the grains of silver thrown down were much more uniformly distributed than at present, most of the trouble would probably vanish. There are at least three possible causes for such an irregular structure as that observed. It may result from a real difference in the sensitiveness of the silver grains, from the lack of uniform transparency of the gelatine covering, or from regions deficient in silver grains. The volume of gelatine in a sensitive emulsion appears to be several times that of the silver grains.¹ Hence it would seem that there must be considerable regions which are devoid of any silver grains an increase in the amount or a further subdivision of silver ought to reduce the scale of the lane structure in the penumbra of star images, with a consequent improvement in their forms and derived positions.

In a following paper² Perrine continues his investigations, studying in detail the effect of an increase in richness of silver in the emulsion and of a diminution in the size of the grain. Seed X-ray plates were chosen on account of their greater richness in silver (about 25 per cent). In addition to ordinary development, the fine-grain development process of Lumière-Seyewetz was employed. Table II summarizes the results obtained. Thus no

TABLE II
RANGES

	Ordinary Development	Fine-Grain Development
Seed X-Ray, ordinary glass	mm .0030 (.0064)	mm .0029 .0040
Seed 27, ordinary glass0044	.0039
Seed 23, ordinary glass0030
Seed 23, plate glass0016
Seed Lantern, ordinary glass0015	.0018
Seed Transparency, plate glass0022	.0020

¹ About nine times in an average emulsion. F.E.R.

² "Results of Some Further Studies on the Structure of Photographic Films and the Effect on Measures of Star Images," *Lick Observatory Bulletin*, No. 178, 1909.

improvement in accuracy due to fine-grain development was found. It is considered that the expected advantage of an increase in richness of silver is borne out. Further conclusions of importance deduced by the author are that thin or weak images show greater discordances than dense or fully exposed ones, and that, when the thickness of the emulsion is reduced, the discordances are also reduced. In addition, "there appears to be a decided tendency . . . for thick films and thin images to give values of the distances systematically too large." He finds:

Plate	Mean Δ in mm
Rapid (thick film)	17.4628
Slow (thin film)	17.4600
Distance on pattern plate	27.4604

The author concludes:

Such an effect could be explained by supposing a swelling action of some sort to take place after the images were impressed upon the film, and during the process of developing, fixing, washing and drying. Just why a swelling should take place is not clear. It would seem more natural to expect a shrinkage, owing to the removal of unaffected silver from the film in fixing.

The subject of film distortion has been investigated by S. Albrecht¹ who covers important points in his investigation not considered by previous workers. To quote:

The most important features of the plan upon which my work was begun were investigations of the effects of (a) the position of the plate during the processes of washing and drying; (b) the rate of drying; (c) abrupt changes in the rate of drying during the process; (d) changes in the position of the plate while drying, (e) hardness. Emulsions on plate glass were also tried.

The results were entirely negative. The conclusions, however, which are of importance and interest, are summarized by the author as follows:

(1) For the size of plate used (4×5 inches) it was found to be entirely indifferent whether the plate be vertical or horizontal during development, fixing, washing and drying. (2) Within the range of the observations, hardener, the rate of drying, and changes in the rate of drying and in the position of the plate during the process of drying introduced no general distortion of the

¹ "On the Distortions of the Photographic Films on Glass," *Astrophysical Journal*, 35, 349, 1907.

gelatine film. (3) Local distortions were found on artificial star plates and on spectrograms. These distortions were confined in each case to an area equal to a small fraction of a square millimeter. The largest lateral displacement found at any point in the distorted area was .020 mm while the great majority were less than one-fourth of this amount. (4) The distortions appear to be principally of two different kinds: one was due to an actual movement of a minute portion of the film, the other was an apparent shift of the image due to the peculiar arrangement of the silver grains or to local differences in the sensitiveness of the film. (5) The results obtained from one plate-glass plate showed no advantages of the plate glass over the ordinary commercial plates in the matter of distortions of the film.

The present writer¹ in 1912 incidentally found an apparently new kind of film distortion. Small plates, 27×37 mm, which were dried in a chimney type of drying-box, showed a large general expansion amounting to one part in twelve hundred. It was further found that if a plate containing star images, in this distorted condition, was soaked in water and dried in the *ordinary* way, the distortion disappeared. In this same investigation a determination was made of the probable error of a single measured distance between reseau lines on plates dried in air and in alcohol respectively. There was a decided difference in favor of the alcohol-dried plates, as the following figures show:

Probable error of a measured distance, air dried $\pm .0020$ mm
Probable error of a measured distance, alcohol dried $.0012$ mm

Uniformity of drying, secured very effectively by immersion in alcohol, is thus seen to be an important factor in reducing film distortion at least on plates of these small dimensions.

In a former paper² the writer has accounted for displacements in position of images which are close together by assuming local inequalities in drying. In the case of an isolated image such a hypothesis does not commend itself unless one assumes a non-homogeneity of structure in the gelatine. This would lead to local variations in the rate of drying. Except for the observations of the writer quoted above, there are no well-authenticated cases of general

¹ *Special Publications, U.S. Coast and Geodetic Survey*, No. 27, p. 44.

² "Image Contraction and Distortion on Photographic Plates," *Astrophysical Journal*, 52, 106, 1920.

expansion or contraction of film on glass. The amount of the observed displacements appears to be constant for all distances. This fact suggests that the phenomenon of apparent displacement is of purely local origin and extent. From this standpoint, namely, of local action, collecting the various suggestions which have been advanced, we have as possible causes:

1. Local drying strains.
2. Local variations in distribution of the silver-bromide grains.
3. Local variations in sensitiveness.
4. Local variations in development.
5. Grain clumping, i.e., graininess or local variations in the distribution of the developed grains.

It is quite evident that much more data must be accumulated before the phenomenon in question can be established beyond dispute. The writer has made a number of experiments and measurements, suggested by various aspects of the subject, which will now be described. In general, the distances measured were chosen small, which makes for greater accuracy, but not so small that the images might possibly be affected by mutual action. Instead of forming artificial star images by contact printing, they were projected on the plate in the precision camera. It was felt that this is much the safer procedure of the two. The only drawback is a variation of focal distance from exposure to exposure due to possible irregularities in the plate. This effect should be small, however, since the plates are only one inch wide and are pressed firmly against a metallic bed in the plate holder. In order to appraise the variation, if present, the pattern plate was made with four artificial stars distributed in a straight line. If there is a variation of focus, the distance of the two outer stars should show a greater probable error. It will be seen that this is not the case.

Experiment A.—Six plates were exposed in the precision camera (reducing 20 times) to the test object containing four artificial stars, *A*, *B*, *C*, *D*, lying on a straight line. Ten exposures were made in rapid succession on each plate. The plate holder slides smoothly and accurately in metallic grooves. The plates were as follows:

1. Fine-grained orthochromatic thin emulsion (called astronomical).

2. Seed 30, medium thickness.

3. Triple-coated orthochromatic.

These plates, of varying emulsion-thickness, were chosen to test the thickness-effect (see Perrine's result, p. 37). Two exposure times were chosen, giving weak or surface images and strong, deep images, respectively. Each plate was measured twenty times, ten direct and ten reversed, on ten separate days. "One measure" in Table III signifies the mean of a single direct and reversed measure, each image being bisected but once. The readings were all started from the same point on the screw, so that secular and periodic screw and bearing errors are eliminated. The mean results are contained in Table III. Magnification used, 75.

TABLE III
MEAN Δ AND PROBABLE ERRORS

PLATES	EMUL- SION THICK- NESS	DIAM- ETER OF IMAGES	$BC = \Delta_1$			$AD = \Delta_2$		
			Mean Δ_1	Probable Errors		Mean Δ_2	Probable Errors	
				One Distance (Photo- graphic)	One Measure (Per- sonal)		One Distance (Photo- graphic)	One Measure (Per- sonal)
Weak (2*) exposure:		mm	mm	mm	mm	mm	mm	mm
Astronomical014	.021	1.3321	.00040	.00043	2.6512	.00085	.00058
Seed 30030	.027	1.3317	.00075	.00063	2.6510	.00134	.00055
Triple C. Ortho060	.024	1.3318	.00084	.00065	2.6520	.00180	.00068
Means			1.3319	.00066	.00057	2.6517	.00133	.00060
Strong (6*) exposure:								
Astronomical014	.061	1.3310	.00046	.00043	2.6510	.00064	.00042
Seed 30030	.067	1.3314	.00072	.00054	2.6513	.00072	.00053
Triple C. Ortho060	.065	1.3324	.00075	.00062	2.6519	.00080	.00049
Means			1.3319	.00064	.00053	2.6514	.00072	.00048

Comparing in this table the means of mean Δ for weak and heavy exposures, it is found that

$$\text{Mean } \Delta \text{ (heavy exposure)} - \text{Mean } \Delta \text{ (weak exposure)} = -.00015 \text{ mm}$$

Comparing mean Δ by plates, agreement is not so marked. However, the pronounced increase in Δ for thick emulsions, found by Perrine, is not in evidence.

Comparing probable errors, the superiority of the thin fine-grain astronomical emulsion is manifest. This plate is by no means slow. It accordingly has distinct advantages in astronomical

work. The photographic probable errors are in general smaller than obtained by previous workers. In fact they are so small as to materially increase faith in the accuracy of photographic registration of position, under properly controlled conditions. However, there is no doubt that occasional, sensible, apparent displacements of images manifest themselves, amounting to .002 mm at the maximum. In order to see if there is any dependence of displacement on the character of the image, all images on the foregoing series of plates concerned in producing large displacements were examined. In every case the images of each pair were round and regular, giving no sign of disturbance. On the other hand distances measured between images which were very irregular in outline—cases where one would expect an apparent displacement—did not indicate anything abnormal. These facts tend to throw doubt on the irregularity-of-outline theory of image displacement and are accordingly more favorable to the theory of local gelatine disturbance, i.e., disturbances confined to minute volumes of gelatine such as would be produced, for example, if the gelatine were not strictly homogeneous. In this case it should be found that local distortions depend, not on emulsion, but on the gelatine and on the thickness of the coating. It would be difficult to explain the large distortions found by Albrecht (p. 38) on any other basis.

The comparatively large value of the photographic error for Δ in the case of weak exposures, shown in Table III, is difficult to explain. That it cannot be due to irregularities in focal distance mentioned above is indicated by the results given in Table IV. Each Δ is the mean of twenty measures on ten days, so that accidental measuring errors are eliminated. Comparing corresponding v 's, there is no evidence of correlation.

The fidelity of the photographic plate in recording distance is primarily dependent upon the physical properties of gelatine, which are peculiar and exceedingly complex. The present form of gelatine is sensitive both to heredity, history, and immediate environment. "Any 'structure' is not inherent in the gelatin, but is an environment impress, a strain structure in the original mass."¹

¹ Sheppard and Elliott, "The Drying and Swelling of Gelatin," *Journal American Chemical Society*, 44, 379, February, 1922.

It is to be pointed out that the form characteristics are not to be regarded as merely two-phase, the wet and dry phase, with intermediate values, for even in the dried condition two or more dimensional forms can exist in the same mass, depending on its previous history. An example of this is recorded on page 38. Numberless experiments suggest themselves which would be useful in the study of the forms of gelatine masses in general and in particular in the study of sheets of gelatine containing emulsions coated on plates. Sheppard and Elliott (*loc. cit.*) have made a partial study of the shapes which drying masses of gelatine take under various condi-

TABLE IV
Δ FOR WEAK EXPOSURES

EXPOSURE	TRIPLE-COATED ORTHO PLATE			
	Δ ₁	τ ₂	Δ ₃	τ ₄
1.....	I.3331	-13	.6492	+28
2.....	.3319	- 1	.6478	+42
3.....	.3325	- 7	.6536	-16
4.....	.3311	+ 7	.6549	-29
5.....	.3329	-11	.6553	-33
6.....	.3295	+23	.6527	- 7
7.....	.3329	-11	.6520	0
8.....	.3323	- 5	.6503	+17
9.....	.3310	+ 8	.6506	+14
10.....	.3309	+ 9	.6540	-20
Means.....	I.3318	2.6520

tions, in particular with one face constrained, which is the case of interest in photography. From their experiments it might be inferred that the upper layer of gelatine on a photographic plate *contracts* on drying, at least at the edges of the plate. It is of interest to make a special study of what happens in this particular and important case. It may not be of practical interest to the astronomer to know what becomes of the photographic image when the film is wet. Knowing that the images on hydration and dehydration move in a general direction perpendicular to the plate, it is only necessary that these opposite movements take place along the same path. This is, of course, an assumption which must be proved. The expansion of the film in development takes place

under entirely different physical and chemical conditions from its contraction on drying, due to the removal of the unexposed silver bromide and to other factors. The natural inference is that distortion would be very likely to take place. This has been the viewpoint of astronomers who have gone to considerable trouble to determine the character and amount of the distortions.

Measurements on wet and dry plates.—In order to observe the effect of swelling, moderately large distances were chosen. Since the effect, if any, should be a maximum on thick films, a triple-coated plate was used. Two groups of images 30 mm apart, symmetrically located on a plate 1×5 inches, were measured. The images were small and confined to the surface, so that any surface creep would be disclosed. The fixing bath contained no hardener. After measurement, dry and wet, the plate was bathed in a hardening (formalin) bath and the measurement repeated. Δ below is the mean of five independent distances, measures being made direct and reversed.

	Mean Δ in mm
Plate measured wet.....	30.1380
Plate measured dry.....	30.1380
After formalin bath, wet.....	30.1368
After formalin bath, dry.....	30.1389

These measures show that even under the extreme conditions chosen, the movements of the images in swelling and in drying are outward and inward along the same path, which path is without a doubt perpendicular to the plate. A very minute effect is shown to have resulted from the hardening bath, but is so small as to be doubtful. The effect of hardening is taken up in more detail on page 46.

It is well known that in the case where images are located near the edge of a plate, measures are unreliable. In order to find out the exact nature of the phenomena exhibited in this case, a very narrow strip, 4.3 mm wide, was cut from a plate containing exposures of the four-hole test object, in such a way that the two outer images, *A* and *D*, were but 0.8 mm from the edge. This plate was then subjected to a series of hydrations and dehydrations and measured at each phase. Table V contains the results. Soakings were for thirty minutes in water at 70° . Very pronounced dis-

tortions are exhibited in this table, which are seen to be considerably less in the case of the images *BC* farther from the edge. It is to be noted that after the first wetting the film always *expands* on drying.

TABLE V
DISTORTIONS OF PLATE

	Δ			
	<i>AD</i>	Increase μ	<i>BC</i>	Increase μ
1. Original, dry.....	2.655	1.333
2. Wet	2.665	+10	1.337	+4
3. Dry	2.702	+47	1.372	+39
4. Wet	2.680	+25	1.345	+12
5. Dry	2.707	+52	1.368	+35
6. Wet	2.685	+30	1.352	+19
7. Dry	2.712	+57	1.374	+41
Distances of images from edge	0.8 mm		1.5 mm	

Further data were obtained from similar measurements on a wider plate (8.1 mm wide). The plate was alternately wetted and dried seven times. The mean results are given in Table VI.

TABLE VI

	<i>AD</i>	<i>BC</i>
Original, dry.....	mm 2.653	mm 1.331
Measured wet.....	2.653	1.333
Measured dry.....	2.659	1.336
Distances of images from edge	2.7	3.4

The same phenomena are exhibited, but to a less extent, as was to be expected, since the distances from the edge are greater.

Cause of edge distortion.—At first sight, it would appear that on drying there must be a shrinkage of the gelatine at the edge of an image taking place in the plane of the plate, following the analogy of a cube of gelatine drying with one face constrained. In the above experiments, however, the opposite was found to be the case, i.e., the gelatine *expanded* on drying. The explanation of the phenomenon appears to be the same as that proposed by the writer

in a former paper¹ in explaining the mutual action of adjacent images. It is imagined that in drying, whenever any differential action occurs such as one portion of the plate drying more quickly than another, there is a migration of gelatine with its encompassed images which takes place in the direction of the region which has dried first.² This principle is manifestly applicable to the edges of plates which dry more quickly than the center. The drying line accordingly creeps in from edge to center, accompanied by migration of gelatine outward. It will be convenient to make use of the term "hydration gradient" which can be defined as the maximum derivative of the specific water content with respect to any direction in the plane of the plate. On account of the gelatine setting very quickly at the extreme edge, leading to a very large hydration gradient, there is greater differential action in this region and accordingly a greater transfer of gelatine. The hydration gradient at any distance from the edge greater than 10 mm, roughly, appears to be so low that transfer of gelatine and consequent distortions are insignificant. In this case the movements are strictly up and down or normal to the plate. It is quite probable that irregularities in the thickness of the film coating lead to uneven drying and consequently to distortions.

Effect of hardener.—A small plate 16×24 mm was cut in such a way that a group of images lay near each long edge in a line perpendicular to it. After measuring dry and wet, the plate was bathed in formalin and remeasured. Table VII contains the results. In the case of Δ_1 , the images are 1.3 mm from the edge of the plate, as indicated in the table.

¹"Image Contraction and Distortion on Photographic Plates," *Astrophysical Journal*, 52, 106, 1920.

²Since the foregoing was written my attention has been called to a paper by H. Stintzing in *Koll. Chem. Beihefte*, 6, 231-96, 1914, entitled "The Influence of Light on Colloid Systems." He finds that it is a general property of colloid systems, of which gelatine is an example, that exposure to radiation produces an increase in concentration of the colloid in the portions insulated, migration taking place from the unaffected regions. The phenomenon only takes place if evaporation is permitted. The author explains the phenomenon as a change in the distribution of the colloid resulting from local inequalities in the rate of evaporation and affirms that this is a general property of colloid systems.

It will be noticed from this table that no effect results from the hardening bath until the plate has dried. The small effect appearing in the fourth line of the table is doubtless due to partial drying before the measures could be completed. After hardening, true distances can no longer be obtained on the plate by soaking in water. Distances are now the same, whether measured wet or dry, and correspond to the false distances indicated in line No. 3 of the table, and not to the true distances of line No. 1. The reason for this is not far to seek. The hardening has destroyed nearly all of the power of the gelatine to swell on immersion in water. In

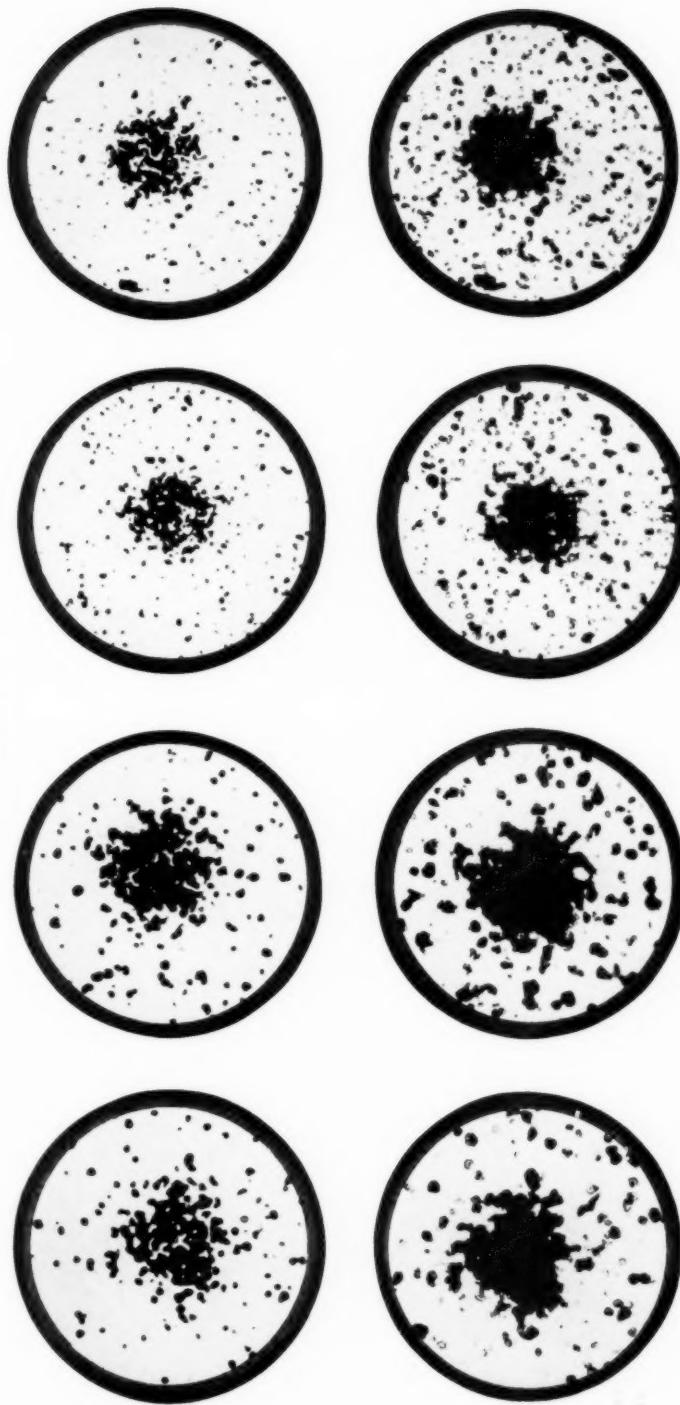
TABLE VII
MEASURES ON HARDENED PLATE

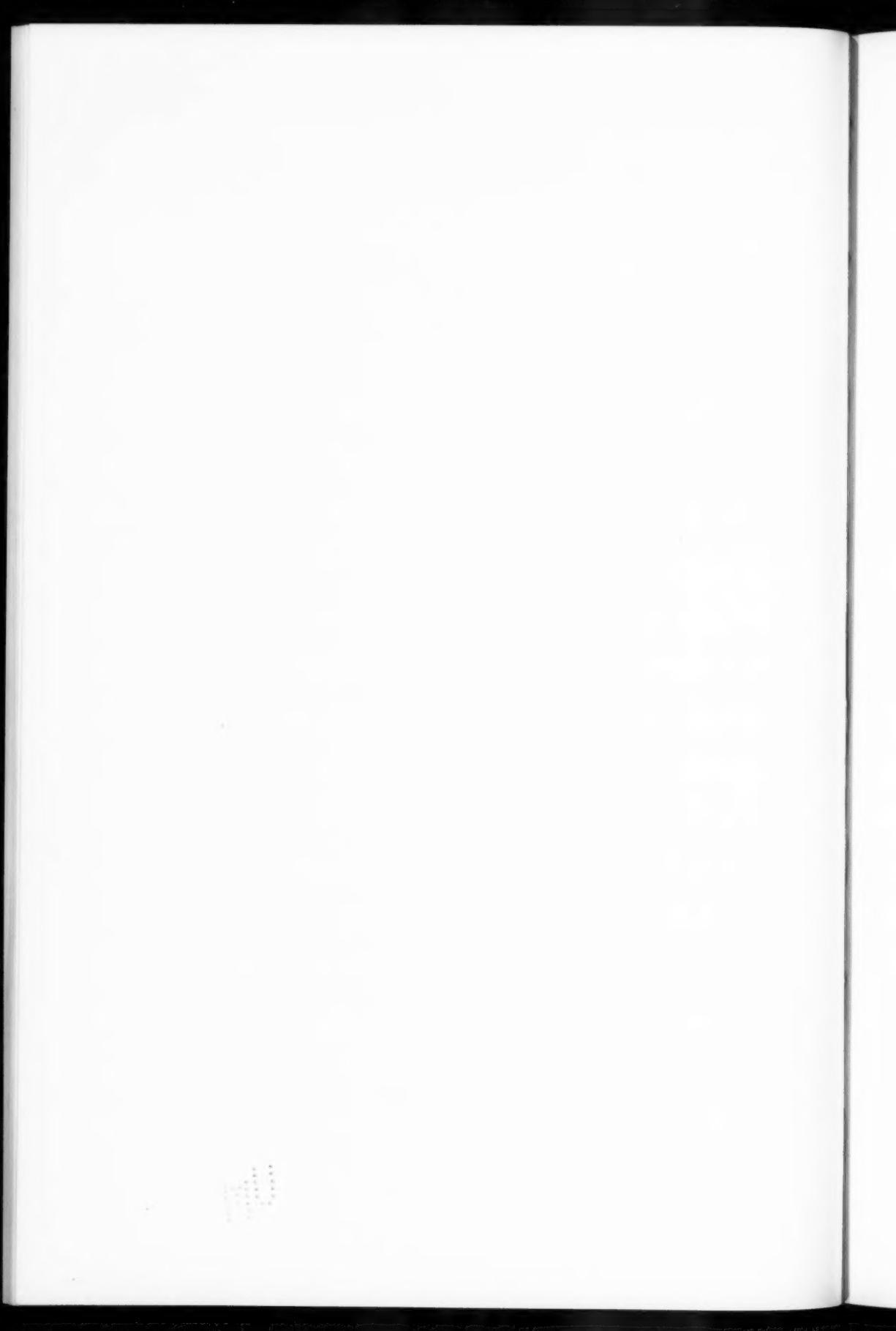
	Δ_1		Δ_2		Δ_3		Δ_4	
	mm	Exp. (μ)						
Original (after cutting), dry.....	14.173	12.863	10.194	8.881
Soaked one hour in water, wet.....	14.173	12.863	10.193	8.877
Dry.....	14.195	+22	12.881	+18	10.204	+10	8.889	+ 8
Soaked one hr. in water, then 15 min in formalin, wet.....	14.180	+ 7	12.866	+ 3	10.195	+ 1	8.880	- 1
Dry.....	14.194	+21	12.871	+ 8	10.199	+ 5	8.883	- 1
Soaked one hr. in water, then 15 min in formalin, wet.....	14.193	+20	12.872	+ 9	10.201	+ 7	8.880	- 1
Dry.....	14.197	+18	12.872	+ 9	10.200	+ 6	8.881	0
Soaked one hr. in water, then 15 min in formalin, wet.....	14.190	+17	12.870	+ 7	10.200	+ 6	8.882	+ 1
Distance of each image from edge of plate	1.3		2.0		3.4		4.0	

the case of the foregoing plate, the swelling was found to be only 40 per cent as compared with an average of 700 per cent on unhardened plates.

Effect of intensification.—Although the present investigation has not disclosed any clear effect upon position measurements which is due to the raggedness of outline of the images measured, there is the possibility of such errors being present. The a priori reasons for the existence of such are strong. If actually present there remains the attractive probability that intensification of the images will lead to improved results. Such improvement should be especially noticeable in the case of weak images in which the defects of structure are more pronounced. Plate I shows a number of

PLATE I





photomicrograms of weak images. In the lower row of Plate I the same images are shown after intensification. Images having an unusual amount of distortion were chosen. One is of an equilateral triangular form. It is to be understood that these irregularities are not due to the optical system but are accidental configurations on the photographic plate. It will be noted that intensification has not altered the larger peculiarities. In the case of the smaller defects which include open spaces, bays, and capes, there is a decided filling in, so that there is a considerable improvement in the appearance of the image after intensification.

In order to make a numerical test of the effect of intensification, a Seed 30 plate containing weak exposures was chosen for test. The system of exposure has been described on page 40. In fact the plate chosen figures in the second line of Table III. Table VIII

TABLE VIII
EFFECT OF INTENSIFICATION

EXPOSURE	AD				BC			
	Before Intensification		After Intensification		Before Intensification		After Intensification	
	mm	v	mm	v	mm	v	mm	v
1.....	2.6505	+14	2.6508	+ 5	1.3297	+20	1.3304	+14
2.....	.6526	- 7	.6518	- 5	.3315	+ 2	.3305	+13
3.....	.6539	-20	.6530	-17	.3308	+ 9	.3302	+16
4.....	.6539	-20	.6525	-12	.3329	-12	.3328	-10
5.....	.6524	- 5	.6521	- 8	.3314	+ 3	.3317	+ 1
6.....	.6540	-21	.6526	-13	.3318	- 1	.3326	- 8
7.....	.6502	+17	.6495	+18	.3324	- 7	.3318	0
8.....	.6483	+36	.6480	+33	.3327	-10	.3326	- 8
9.....	.6510	+ 9	.6496	+17	.3310	+ 7	.3310	+ 8
10.....	.6525	- 6	.6533	-20	.3332	-15	.3347	-29
Means..	2.6519	2.6513	1.3317	1.3318

contains the results of measurement of this plate before and after intensification. Each Δ is the mean of ten measurements made on as many days. Comparison of the v 's before and after intensification in this table proves that intensification has had no effect on the apparent occasional displacements of star images, which are indicated by abnormal values of v . On account of the improve-

ment in the appearance of intensified images noted above, it might be expected that the accidental measuring errors are reduced. That this is the case is shown by Table IX. Accordingly, the only gain

TABLE IX
PROBABLE ERROR OF A SINGLE MEASUREMENT
(Mean of Direct and Reversed)

	Δ_1	Δ_2
Before intensification.....	mm $\pm .00055$	mm $\pm .00063$
After intensification.....	.00044	.00050

on intensification appears to be a diminution in the error of measurement. The constant errors in position of the image are not affected.

ROCHESTER, NEW YORK
August 22, 1922

INVESTIGATIONS ON PROPER MOTION

NINTH PAPER: INTERNAL MOTION IN THE SPIRAL NEBULA
MESSIER 63, N.G.C. 5055¹

By ADRIAAN VAN MAANEN

ABSTRACT

Spiral Nebula M 63 (N.G.C. 5055).—Comparison of two plates taken in 1910 and 1922 by Ritchey and Humason, respectively, gives, with respect to 21 comparison stars, an annual *proper motion* for the nebula as a whole of $\mu_a = +0.^{\circ}005$, $\mu_\delta = -0.^{\circ}015$. The motions of 98 nebular points with reference to the nebula are tabulated and also shown on the *photograph*, Plate II. The *internal motion* is found to be a stream motion of $+0.^{\circ}019$ outward along the arms of the spiral, combined with an outward transverse motion of $+0.^{\circ}004$. There is a slight increase in the motions with increasing distance from the center. The results are analogous to those found for M 33, 51, 81, 94, 101, and N.G.C. 2403, but are even more convincing because of the long time-interval between plates and the many nearly starlike points.

Curtis² describes this object as “a bright, beautiful spiral $8' \times 3'$ in p.a. 98° . Has an almost stellar nucleus. The whorls are narrow, very compactly arranged, and show numerous almost stellar condensations.”

Two plates taken at the 25-foot focus of the 60-inch reflector were measured with the new stereocomparator: the old plate was secured by Mr. Ritchey on March 9, 1910, the new plate by Mr. Humason on May 28 and 29, 1922; both plates have an exposure of five hours and are of good quality.

The plates were measured in four positions, east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. Twenty-one comparison stars and 98 points, the latter presumably belonging to the nebula, were measured. The measures in right ascension were combined into one set, and those in declination into another; then the measured quantities were multiplied by 0.698 to reduce the values expressed in parts of the micrometer screw to annual motions in thousandths of a second of arc. These quantities, m_a and m_δ , respectively, were used as the first members in equations of condition of the form:

$$\left. \begin{aligned} m_a &= a + bx + cy + dx^2 + exy + fy^2 + \mu_a \\ m_\delta &= a' + b'x + c'y + d'x^2 + e'xy + f'y^2 + \mu_\delta \end{aligned} \right\} \quad (1)$$

¹ Contributions from the Mount Wilson Observatory, No. 255.

² Lick Observatory Publications, 13, 34, 1918.

in which $a \dots f$, $a' \dots f'$ are the plate constants, x and y the co-ordinates in right ascension and declination, and μ_a and μ_δ the annual proper motions. By a least-squares solution the plate constants were determined from two sets of equations of the form (1), yielded by the 21 comparison stars. These constants were substituted into equations of the form (1) for all objects measured, thus giving μ_a and μ_δ , the components of the motion with respect to the mean of the comparison stars. These quantities are listed in the fourth and fifth columns of Table I; the second and third columns give the positions with respect to the center of the nebula, accurate to a tenth of a minute of arc.

In order to derive the internal motions of the nebula, the values of μ_a and μ_δ must be freed from the motion of the nebula as a whole. The same procedure was used as for M 33, 51, 81, 94, 101, and N.G.C. 2403.

a) The mean motion of the 98 points is

$$\mu_a = +0.^{\prime\prime}005, \quad \mu_\delta = -0.^{\prime\prime}015.$$

b) Combining the mean motions in quadrants I and III, we have

$$\mu_a = +0.^{\prime\prime}002^5, \quad \mu_\delta = -0.^{\prime\prime}015;$$

while for quadrants II and IV

$$\mu_a = +0.^{\prime\prime}008, \quad \mu_\delta = -0.^{\prime\prime}015.$$

All four quadrants combined give

$$\mu_a = +0.^{\prime\prime}005, \quad \mu_\delta = -0.^{\prime\prime}015.$$

c) Using only the 80 points closest to the center, which have a more symmetrical distribution, we find for quadrants I and III, $\mu_a = +0.^{\prime\prime}003$, $\mu_\delta = -0.^{\prime\prime}014^5$; while for quadrants II and IV, $\mu_a = +0.^{\prime\prime}005^5$, $\mu_\delta = -0.^{\prime\prime}014$. For all four quadrants $\mu_a = +0.^{\prime\prime}004$, $\mu_\delta = -0.^{\prime\prime}014$.

For the motion of the nebula as a whole, the mean result from the three methods is

$$\mu_a = +0.^{\prime\prime}005, \quad \mu_\delta = -0.^{\prime\prime}015.$$

TABLE I
CO-ORDINATES AND MEASURED ANNUAL DISPLACEMENTS

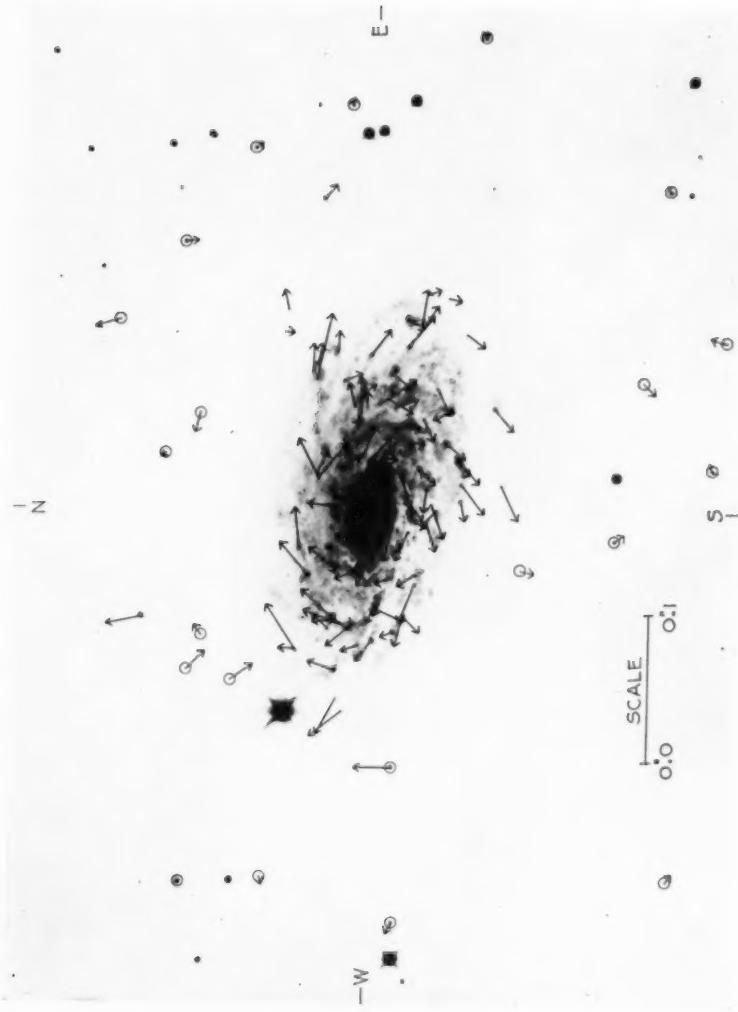
No.	<i>x</i>	<i>y</i>	μ_a	μ_d
a.....	-6.3	+3.2	0.000	0.000
b.....	-6.2	+1.8	- 5	- 2
c.....	-2.9	+2.4	+ 10	- 15
d.....	-2.7	+3.1	+ 6	- 12
e.....	-2.1	+2.9	+ 6	+ 5
f.....	-7.0	-0.5	- 7	+ 3
g.....	-4.3	-0.5	0	+ 26
h.....	-6.2	-5.2	+ 5	- 4
i.....	-1.0	-2.6	- 2	- 9
j.....	-0.5	-4.2	+ 4	- 7
k.....	+0.8	-5.9	- 1	+ 1
l.....	+2.3	-4.7	- 8	- 8
m.....	+3.0	-6.1	+ 3	+ 11
n.....	+5.5	-5.0	- 1	+ 1
o.....	+8.1	-1.9	+ 3	+ 2
p.....	+7.0	+0.5	+ 4	- 1
q.....	+6.2	+2.1	+ 3	- 5
r.....	+4.6	+3.3	+ 1	- 9
s.....	+3.2	+4.3	- 5	+ 17
t.....	+1.7	+3.0	- 13	+ 3
u.....	+1.0	+3.5	- 3	+ 3
I.....	-3.4	+0.5	- 12	+ 7
2.....	-3.2	+0.6	- 19	- 2
3.....	-2.7	+0.6	+ 10	+ 3
4.....	-2.4	+1.3	+ 36	+ 6
5.....	-2.3	+1.3	+ 5	- 5
6.....	-2.3	+0.2	+ 1	- 4
7.....	-2.2	-0.1	- 9	- 4
8.....	-2.1	-0.4	+ 1	- 7
9.....	-2.0	+0.4	+ 15	- 1
10.....	-1.9	+0.5	+ 13	+ 10
11.....	-1.9	+0.4	+ 5	- 17
12.....	-1.9	+0.3	- 11	+ 3
13.....	-1.9	-0.5	- 12	- 12
14.....	-1.9	-0.6	+ 13	+ 5
15.....	-1.8	-0.6	- 6	- 26
16.....	-1.9	+0.7	+ 4	- 3
17.....	-1.8	+0.7	+ 23	+ 4
18.....	-1.6	-0.1	0	- 13
19.....	-1.6	-0.1	- 8	- 10
20.....	-1.3	-0.8	- 21	- 5
21.....	-1.2	0.0	+ 1	- 12
22.....	-1.1	-0.4	+ 1	- 3
23.....	-1.1	-0.8	- 3	- 2
24.....	-1.1	+1.1	+ 26	+ 4
25.....	-1.0	+0.1	- 2	+ 5
26.....	-1.0	+0.7	+ 18	+ 2
27.....	-0.9	+0.7	- 4	- 18
28.....	-0.9	0.0	- 2	- 14
29.....	-0.8	-0.1	- 1	- 2
30.....	-0.8	0.0	- 8	- 8
31.....	-0.7	0.0	- 17	- 6
32.....	-0.7	+0.3	+0.016	-0.002

TABLE I—Continued

No.	x	y	μ_a	μ_b
33.	-0.7	+1.2	+0.033	-0.012
34.	-0.6	-0.5	- 1	- 11
35.	-0.4	-0.7	- 14	- 6
36.	-0.3	-0.2	- 6	- 13
37.	-0.3	-0.4	- 18	- 10
38.	-0.2	-0.8	+ 3	- 22
39.	-0.2	-1.0	- 15	- 21
40.	+0.2	-0.7	- 6	- 17
41.	-0.1	+0.7	+ 19	- 18
42.	+0.1	+0.7	+ 6	+ 1
43.	0.0	-1.1	- 5	- 5
44.	+0.1	-1.0	- 17	- 27
45.	+0.3	-0.6	- 12	- 27
46.	+0.2	-1.6	- 8	- 17
47.	+0.4	-1.0	- 11	- 13
48.	+0.5	-2.2	- 19	- 26
49.	+0.5	-1.6	- 16	- 30
50.	+0.6	-0.3	- 5	- 16
51.	+0.6	-0.4	+ 4	- 5
52.	+0.6	-0.6	- 15	- 20
53.	+0.7	-0.9	- 5	- 25
54.	+0.7	+0.5	+ 15	- 27
55.	+0.6	+0.9	+ 31	- 1
56.	+0.7	+1.0	+ 20	- 22
57.	+0.7	+0.8	+ 23	- 30
58.	+0.9	-1.5	- 4	- 24
59.	+0.9	-1.5	- 12	- 29
60.	+1.0	-1.5	+ 1	- 19
61.	+0.9	-0.3	+ 4	- 23
62.	+0.9	+0.1	+ 21	- 30
63.	+1.0	+0.2	+ 25	- 24
64.	+1.1	+0.5	+ 26	- 24
65.	+1.2	-1.3	- 5	- 28
66.	+1.3	-0.5	+ 2	- 33
67.	+1.4	-0.4	+ 6	- 33
68.	+1.5	+0.3	+ 36	- 18
69.	+1.6	+0.2	+ 21	- 17
70.	+1.6	-0.9	- 10	- 22
71.	+1.8	-2.1	- 10	- 27
72.	+1.7	-1.4	- 1	- 4
73.	+1.8	-0.4	+ 11	- 30
74.	+1.8	+0.1	+ 15	- 15
75.	+1.8	+0.3	+ 21	- 6
76.	+1.9	-0.2	+ 21	- 39
77.	+2.0	-0.3	- 4	- 30
78.	+2.0	+0.3	+ 14	- 27
79.	+2.1	-1.1	- 12	- 23
80.	+2.2	+0.5	+ 10	- 28
81.	+2.3	+1.0	+ 48	- 25
82.	+2.4	+1.0	+ 24	- 15
83.	+2.5	+1.0	+ 18	- 18
84.	+2.4	-0.4	- 6	- 28
85.	+2.7	+0.1	+ 23	- 28
86.	+2.8	+0.6	+0.018	-0.016



PLATE II



INTERNAL MOTIONS IN MESSIER 63

The arrows indicate the direction and magnitude of the annual motions. Their scale ($0.^{\circ}1$) is indicated on the illustration. The scale of the nebula is 1 mm = $7.^{\circ}7$. The comparison stars are inclosed in circles.

TABLE I—Continued

No.	x	y	μ_x	μ_y
87.....	+2.8	-0.6	+0.027	-0.033
88.....	+3.0	-0.8	+ 24	- 29
89.....	+3.1	+1.5	+ 5	- 21
90.....	+3.0	-1.6	- 4	- 27
91.....	+3.3	-0.9	+ 26	- 18
92.....	+3.3	-0.6	- 2	- 28
93.....	+3.7	-1.3	+ 3	- 24
94.....	+3.7	-0.9	+ 8	- 24
95.....	+3.4	+1.5	+ 18	- 13
96.....	+5.3	+0.9	+ 14	- 23
97.....	-1.8	+3.9	0	+ 9
98.....	-2.0	+0.7	+0.015	-0.004

Subtracting these values from the annual motions given in Table I, we derive the internal motions, which are given in the second and third columns of Table II. These motions are plotted in Plate II. For the comparison stars, which are surrounded by circles, the motions of Table I are used. The scale of the motions is indicated in the lower left-hand corner. The arrows represent the motions during an interval of about 1500 years. From the results derived for the spirals previously measured, it seems that the internal motions represent a motion along the spiral arms and outward.¹ In order to derive the stream and transverse components of the motions in N.G.C. 5055, we must take into account the fact that this nebula is much inclined to the plane perpendicular to the line of sight.

It is therefore necessary to correct the motions accordingly; in order to do this Mr. Sinclair Smith has by means of two cylindrical lenses constructed a photograph of Plate II, showing the nebula as a circular object, which it presumably is. From this photograph have been read the stream and transverse components given in the fourth and fifth columns of Table II. For points Nos. 89, 95, 96, and 97, in whose neighborhood the streamers could not be traced, this was impossible and they have been omitted in the discussion. The result is: mean stream motion, $+0.019 \pm 0.001$; mean trans-

¹ *Mt. Wilson Contr.*, No. 243; *Astrophysical Journal*, 56, 208, 1922.

TABLE II
ANNUAL INTERNAL MOTIONS

No.	μ_a	μ_b	Stream	Transverse
I.....	-0.017	+0.022	+0.042	-0.004
2.....	- 24	+ 13	+ 28	+ 7
3.....	+ 5	+ 18	+ 38	- 2
4.....	+ 31	+ 21	+ 55	- 7
5.....	0	+ 10	+ 20	+ 8
6.....	- 4	+ 11	+ 22	+ 1
7.....	- 14	+ 11	+ 22	+ 9
8.....	- 4	+ 8	+ 17	- 4
9.....	+ 10	+ 14	+ 30	- 3
10.....	+ 8	+ 25	+ 44	- 9
11.....	0	- 2	- 5	- 1
12.....	- 16	+ 18	+ 26	+ 17
13.....	- 17	+ 3	+ 17	+ 7
14.....	+ 8	+ 20	+ 18	- 37
15.....	- 11	- 11	- 5	+ 24
16.....	- 1	+ 12	+ 16	+ 6
17.....	+ 18	+ 19	+ 41	0
18.....	- 5	+ 2	+ 5	+ 3
19.....	- 13	+ 5	+ 13	+ 5
20.....	- 26	+ 10	+ 30	+ 3
21.....	- 4	+ 3	+ 8	+ 3
22.....	- 4	+ 12	+ 14	- 15
23.....	- 8	+ 13	+ 14	- 19
24.....	+ 21	+ 19	+ 42	+ 19
25.....	7	+ 20	+ 36	0
26.....	+ 13	+ 17	+ 33	+ 8
27.....	- 9	- 3	- 13	+ 1
28.....	- 7	+ 1	+ 3	+ 7
29.....	- 6	+ 13	+ 24	- 2
30.....	- 13	+ 7	+ 18	+ 8
31.....	- 22	+ 9	+ 14	+ 20
32.....	+ 11	+ 13	+ 27	+ 4
33.....	+ 28	+ 3	+ 30	- 2
34.....	- 6	+ 4	+ 6	- 5
35.....	- 19	+ 9	+ 18	- 12
36.....	- 11	+ 2	+ 11	+ 2
37.....	- 23	+ 5	+ 24	- 1
38.....	- 2	- 7	+ 7	+ 12
39.....	- 20	- 6	+ 20	+ 12
40.....	- 11	- 2	+ 23	+ 3
41.....	+ 14	- 3	+ 14	- 5
42.....	+ 1	+ 16	+ 1	+ 32
43.....	- 10	+ 10	+ 9	- 16
44.....	- 22	- 12	+ 25	+ 16
45.....	- 17	- 12	+ 22	+ 22
46.....	- 13	- 2	+ 13	+ 6
47.....	- 10	+ 2	+ 16	- 4
48.....	- 24	- 11	+ 23	+ 24
49.....	- 21	- 15	+ 26	+ 27
50.....	- 10	- 1	+ 9	- 5
51.....	- 1	+ 10	- 16	- 8
52.....	- 20	- 5	+ 25	+ 3
53.....	-0.010	-0.010	+0.019	+0.015

TABLE II—Continued

No.	μ_a	μ_b	Stream	Transverse
54.....	+0.010	-0.012	+0.018	-0.012
55.....	+ 26	+ 14	+ 17	+ 34
56.....	+ 15	- 7	+ 17	- 5
57.....	+ 18	- 15	+ 31	- 22
58.....	- 9	- 9	+ 13	+ 18
59.....	- 17	- 14	+ 25	+ 23
60.....	- 4	- 4	+ 6	+ 5
61.....	- 1	- 8	+ 16	+ 1
62.....	+ 16	- 15	+ 24	+ 16
63.....	+ 20	- 9	+ 18	+ 15
64.....	+ 21	- 9	+ 23	+ 5
65.....	- 10	- 13	+ 21	+ 19
66.....	- 3	- 18	+ 31	+ 14
67.....	+ 1	- 18	+ 32	+ 7
68.....	+ 31	- 3	+ 16	+ 27
69.....	+ 16	- 2	+ 6	+ 15
70.....	- 15	- 7	+ 20	- 5
71.....	- 15	- 12	+ 22	+ 19
72.....	- 6	+ 11	- 12	- 16
73.....	+ 6	- 15	+ 27	+ 8
74.....	+ 10	0	0	+ 11
75.....	+ 16	+ 9	+ 3	+ 19
76.....	+ 16	- 24	+ 41	+ 16
77.....	- 9	- 15	+ 30	- 7
78.....	+ 9	- 12	+ 19	- 1
79.....	- 17	- 8	+ 25	- 5
80.....	+ 5	- 13	+ 15	- 15
81.....	+ 43	- 10	+ 45	- 1
82.....	+ 19	0	+ 18	- 7
83.....	+ 13	- 3	+ 15	- 1
84.....	- 11	- 13	+ 22	- 18
85.....	+ 18	- 13	+ 26	+ 8
86.....	+ 13	- 1	+ 11	+ 9
87.....	+ 22	- 18	+ 31	+ 17
88.....	+ 19	- 14	+ 24	+ 18
89.....	0	- 6
90.....	- 9	- 12	+ 22	+ 8
91.....	+ 21	- 3	+ 3	+ 21
92.....	- 7	- 13	+ 21	- 11
93.....	- 2	- 9	+ 17	+ 1
94.....	+ 3	- 9	+ 9	+ 4
95.....	+ 13	+ 2
96.....	+ 9	- 8
97.....	- 5	+ 24
98.....	+0.010	+0.011	+0.023	-0.002

verse motion, $+0.^{\circ}004 \pm 0.^{\circ}001$. The change in these components with distance (r) from the center is given in Table III.

These results are analogous to those found for M 33, 51, 81, 94, 101, and N.G.C. 2403. Since N.G.C. 5055 has many nearly star-

like points and the quality of the plates is very good, and since the interval between the photographs is longer than for any of the other

TABLE III

<i>r</i>	Stream	Transverse	<i>n</i>
o'-1'	+o".016	+o".004	11
1-2	+o.018	+o.005	36
2-3	+o.020	+o.002	32
3-4	+o.022	+o.006	12
4-5	+o.021	+o.015	3

spirals measured on plates taken with the 60-inch reflector, the results give a strong corroboration of the internal motions in these objects.

MOUNT WILSON OBSERVATORY

October 1922

A PARTIAL EXPLANATION, BY WAVE-LENGTHS, OF THE K-TERM IN THE B-TYPES. II

By SEBASTIAN ALBRECHT

ABSTRACT

K-term of radial velocity for Class B stars.—In a way similar to that in which the K-term had previously been found to be in part due to erroneous normal wave-lengths for the oxygen and nitrogen lines, it is shown to be also in part due to erroneous normal wave-lengths for the silicon lines 4552, 4567, and 4575 and for the helium blend 4713. On the basis of new wave-lengths for oxygen and nitrogen, silicon, and helium 4713, the total reduction in the K-term would amount to about 2 km for classes B₀ to B₂, 1 km for B₃, and 0.3 km for B₅ to B₈.

Fundamental radial velocities in Class B.—A discussion of the causes or reality of the K-term involves first of all a discussion of fundamental radial velocities. Radial velocities of the Class B stars contain the inherent weakness that some of the lines which were most extensively employed are double, with unequal components, and therefore not suited for fundamental work. For such work the single lines, principally those of oxygen, nitrogen, and silicon, will have to be relied upon, and these require, as a preliminary, additional laboratory study.

No one realizes better than the radial velocity observer himself the misleading nature of the precision attained in radial velocities as derived from the internal agreement in the determinations. A precision of a tenth of a kilometer, in the absolute sense, is illusory. In a note in the *Astrophysical Journal* in 1910 Professor Frost raised the question as to whether we really know the radial velocity of any star in the heavens to the nearest kilometer. In view of the great advances in this line of work made at the Lick, Yerkes, and other observatories, the above statement seemed ultra-conservative. However, we now know of large systematic errors pervading all radial velocities, amounting in classes B and M to four times the limit of 1 km postulated by Frost.

The origin of these systematic errors, now commonly known as the K-term, is not well understood, though various possible causes, including the Einstein effect, have been proposed. It has been shown¹ that they are at least in part due to inaccuracies in the laboratory values of the wave-lengths which were employed. Recent laboratory wave-lengths for twenty oxygen and nitrogen lines occurring in B-type stars, are systematically 0.063 Å longer than the adopted normals. This corresponds to a change of

¹ *Astrophysical Journal*, 55, 361, 1922.

-4.2 km per second in the radial velocities derived from them. A discussion of the available portions of the data on which the K-term is based, indicated that the new wave-lengths for this one group of lines alone would reduce the K-term by about 0.3 km for the entire B class and by about 0.8 km for classes Bo to B₂.

Besides showing that an additional portion of the K-term is due to inaccurate normal wave-lengths, I wish to point out what seems, at least for the present, a more or less inherent weakness in the radial velocities of the B-types and to emphasize especially the need of further laboratory study of the lines employed.

For the silicon and helium lines new laboratory wave-lengths are available. Table I gives the recent and the more important of

TABLE I
LABORATORY WAVE-LENGTHS OF SILICON AND CORRESPONDING CHANGES
IN THE RADIAL VELOCITIES

EXNER & HASCHEK* (USED BY F. & A.)		FROST & BROWN†		CROOKES‡ (NEARLY PURE Si)		SAWYER & PATON§ (VACUUM SPARK)	
λ	Change in v_s	λ	Change in v_s	λ	Change in v_s	λ (Accuracy 0.1 ±)	Change in v_s
	km		km		km		km
4552.75	o	4552.64	+7.2	4552.841	-6.0	4552.74	+0.7
4567.95	o	4567.90	+3.3	4568.123	-11.4	4567.82	+8.5
4574.9	o	4574.79	+7.2	4574.823	+5.0	4574.81	+5.9
Mean.....	o	+5.9	-4.1	+5.0

* *Astrophysical Journal*, 12, 49, 1900.

† *Ibid.*, 22, 159, 1905.

‡ *Proceedings of the Royal Society, A*, 90, 512, 1914.

§ *Physical Review*, 19, 256, 1922.

the earlier laboratory wave-lengths of silicon. Sawyer and Paton claim an accuracy of only about 0.1 Å, which is insufficient for our purposes. Crookes, emphasizing the necessity of employing samples of silicon of the highest purity, aimed to attain a high accuracy. If we apply the wave-lengths of Crookes—with the reservation that these results are not final, as will be shown below—to the available radial velocity data¹ we obtain the changes shown in the columns "Si (Crookes)" in Table II.

For helium, only the lines at 4713 are appreciably changed in wave-length. Merrill's² values, reduced to the Rowland

¹ *Publications of the Yerkes Observatory*, 2, 143, 1904.

² *Bureau of Standards Bulletin*, 14, 162, 1917.

system,¹ are 4713.325, intensity 3, and 4713.548, intensity 1. Combined in the usual manner according to intensities the blend (.381) is 0.073 Å longer than the normal value which was employed. Radial velocities obtained from this line are thus changed systematically by -4.6 km per second. The actual changes in stellar radial velocities are illustrated in the columns "4713 (Merrill)" in Table II.

TABLE II
CHANGES IN RADIAL VELOCITY DUE TO NEW WAVE-LENGTHS AND
WEIGHTED PERCENTAGES OF LINES AFFECTED.
MEASURES BY FROST AND ADAMS

STAR	TYPE	O. & N. (CLARK)		Si (CROOKES)		4713 (MERRILL)		TOTAL	
		Change in Radial Velocity	Per Cent of Lines						
ε Ori.....	B	km -0.1	4	km -1.1	13	km -0.5	11	km -1.7	28
ζ Ori.....	B	-0.2	11 0 0	-0.2	11
κ Ori.....	B	-0.2	14	-1.8	26 0	-2.0	40
ξ Per.....	B1	-0.7	32	-1.8	33	-0.2	4	-2.7	69
β CMa.....	B1	-1.0	43	-2.1	33 0 0	-3.1	76
ε CMa.....	B1	-0.7	32	-1.9	33 0 0	-2.6	65
γ Peg.....	B2	-0.2	20	-2.0	28	-0.0	1	-2.2	49
ζ Cas.....	B2	-0.1	7	-1.6	26	-0.1	2	-1.8	35
γ Ori.....	B2	-0.4	11	-1.7	20	-0.2	5	-2.3	36
102 Her.....	B2	0.0	5	-2.0	30 0 0	-2.0	35
ι Her.....	B3 0 1	-1.1	16	-0.5	10	-1.6	26
η Lyr.....	B3	0.0	1	-0.6	8 0 0	-0.6	9
ε Cas.....	B5 0 0 0 0 0 0 0 0
τ Her.....	B5 0 0 0 0 0 0 0 0
ζ Dra.....	B5	-0.1	4 0 0 0 0	-0.1	4
67 Oph.....	B5 0 6	-0.9	18	-0.3	7	-1.2	25
ε Del.....	B5	0.0	6 0 0 0 0	0.0	6
β Ori.....	B8 0 0	-0.2	2	-0.1	3	-0.3	5
λ Crv.....	B8 0 0 0 0 0 0 0 0

The total changes in the radial velocities, due to the new wave-lengths of oxygen and nitrogen, silicon, and 4713 are given in the second last column of Table II. If these wave-lengths prove to be essentially correct systematically, they will eliminate actually about one-half of the K-term in classes Bo to B₃, and less in classes B₅ to B₈.

The laboratory results for silicon are not at all satisfactory. They differ widely from each other, both systematically and inter-

¹ The curve in *Astrophysical Journal*, 41, 347, 1915, is satisfactory for present purposes.

nally. Table III gives a comparison of the laboratory wave-lengths (see Table I) with the corresponding stellar wave-lengths. As the systematic error of the latter cannot at present be safely approximated, this comparison is not competent to furnish a decision in regard to the absolute systematic errors of the different laboratory results. The third items in column 1 of Table III constitute the systematic adjustments of the individual sets of laboratory wave-lengths to the observed stellar wave-lengths, and thus readily yield the mutual systematic laboratory discordances. The total range of 0.153 Å is equivalent to a systematic difference of 10 km per second in radial velocity. In so far as the stellar wave-lengths may serve as a guide, the wave-lengths of Exner and Haschek have

TABLE III
SHOWING THE INTERNAL AGREEMENT OF THE LABORATORY WAVE-LENGTHS
OF SILICON WITH THE WAVE-LENGTHS IN STARS

$\lambda_{\text{Laboratory}} - \lambda_{\text{Stars}}$ — Syst. Dif.	4552.762*	5467.967*	4574.918*
$\lambda_{\text{Exner and Haschek}} - \lambda_{\text{Stars}} + .016 \dots \dots \dots$	+ .004	- .001	- .002
$\lambda_{\text{Frost and Brown}} - \lambda_{\text{Stars}} + .106 \dots \dots \dots$	- 0.16	+ .039	- .022
$\lambda_{\text{Crookes}} - \lambda_{\text{Stars}} - .047 \dots \dots \dots$	+ .032	+ .109	- .142
$\lambda_{\text{Sawyer and Paton}} - \lambda_{\text{Stars}} + .092 \dots \dots \dots$	+ .070	- .055	- .016

* These are the observed wave-lengths in stars. *Publications of the Astronomical and Astrophysical Society of America*, 2, 72, 1911.

apparently exceedingly small accidental errors. This is, however, largely fortuitous as Exner and Haschek published their wave-lengths to only the nearest tenth or hundredth of an angstrom. The accidental errors for the measures of Frost and Brown appear to be moderate, while for the measures of Sawyer and Paton and especially of Crookes they seem to be large.

The lines which are available in Class B stars in the region of spectrum generally employed with three prisms are given in Table IV. It will be noticed that of the lines which were most extensively employed, the carbon line 4267, magnesium 4481, and the helium lines 4472 and 4713 are double with unequal components, and therefore not suited for primary radial velocity work. H γ also is not entirely satisfactory. For the helium line 4438 results in stars are somewhat erratic. For silicon the laboratory wave-lengths are erratic and mutually discordant. Except for the helium

line 4388, the oxygen and nitrogen lines alone seem free from serious objection, and these would give practically no K-term.

Speculation on the probable causes of the K-term cannot be profitable so long as we have not eliminated the possibility that this term has been artificially introduced by the use of erroneous normal wave-lengths. In other words the reality of the K-term is inseparably connected with the problem of determining fundamental

TABLE IV
LINES IN B-TYPE STARS

Carbon	Hydrogen	Magnesium	Helium	Silicon (Crookes)	Oxygen and Nitrogen (Clark)	Metallic Lines and Blends
4267.301 Double: separation o.26 A; intensities 1 and 2	4340.634 Double; separation o.012 A in lab. spectrum. Intensities 7 and 10. Also, enh. Bismuth at 0.7	(4352.083) Rarely measured in stars	4388.100 inten. 3	4552.841 4568.123 4574.823 Lab. val- ues are erratic. Results in stars erratic	4317.327 19.814 45.738 48.246 49.602 51.443 67.074 4415.057 17.143 4447.205 4591.158 96.365 4621.582 30.729 39.043 42.006 49.327 51.032 61.829 4676.426	Measur- able only in late B-types

radial velocities. Table IV brings out well the seemingly inherent difficulties and the urgent need for additional laboratory study to make possible determinations of fundamental radial velocity for the Class B stars. As is well known, blends of unequal components are subject to apparent shifts depending upon the quality of the spectrograms. For a few of the brighter stars the double lines of carbon, magnesium, and helium can probably be resolved and the individual components employed by the use of sufficiently high

dispersion. It would be possible to determine empirically for each spectrograph, by means of laboratory and stellar spectra of the same and higher dispersion, close approximations to the true normal wave-lengths of the blends for each of several standard conditions of intensity, contrast, and grain of the individual regions on the spectrograms. Although such methods give promise of improving our radial velocities in a systematic sense, they can give only an approximation to really fundamental results. Possible differences in the relative intensities of the components, from star to star or from type to type, would produce apparent shifts of the blends and thus introduce systematic errors in the radial velocities. For fundamental results the principal reliance must apparently be placed on the single lines, those of oxygen and nitrogen, of silicon—when the laboratory discordances shall have been removed—of the helium line 4388 (and perhaps 4437), and of the metals when available. On account of its unusual importance in these spectra, it is to be hoped that the use of H γ may also prove feasible. Although this line is double, the separation of the components is only about 0.012 Å, and the enhanced bismuth companion may be absent or inappreciable in types B₅ and earlier.

For purposes of fundamental radial velocities it is highly desirable to have at least one, and preferably two, additional laboratory determinations of wave-length for the oxygen and nitrogen lines. More urgent, however, is the thorough study of the spectrum of silicon in order that the large systematic and internal discordances may be removed. The large relative discordances between the individual lines in the laboratory determinations are apparently not duplicated in stellar spectra, the internal agreement from star to star and from type to type being fairly good. The work of Crookes and of Sawyer and Paton should be repeated with silicon of high purity and with the spark terminals in air and in a vacuum. As the physical conditions in the stars and in the laboratory spark are probably very dissimilar, it would be well to vary the laboratory conditions in whatever way this may be possible.

DUDLEY OBSERVATORY

ALBANY NEW YORK

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REVIEWS

Light, A Textbook for Students Who Have Had One Year of Physics.

By H. M. REESE. Columbia: Missouri Book Company, 1921. Pp. 295. Figs. 139. \$3.50.

Those who are responsible for giving the second courses in light in the universities have long felt the need for a more suitable text. There are many books available, but the majority present far too much material for the time which is allowed. The English texts, particularly, assume a much more general acquaintance with mathematics, even where the calculus is expressly excluded, than our junior students commonly possess. A book of the character of this one is, therefore, very welcome. In some 285 pages, it covers all the ordinary topics in sufficient detail so that the phenomena become familiar and are fitted into their places in the theory.

The book starts with a chapter on the measurement of the velocity of light, where the well-known experiments are covered in slightly more detail than in the ordinary sophomore textbook. It is to be regretted that improvements Michelson introduced in the Foucault-Fizeau method are not described even in principle, in spite of the reviewer's experience that very few junior students have understood them when they have been presented.

Chapter ii inverts the usual order to consider dispersion by a prism before discussing deviation in any detail. This is for the purpose of considering color phenomenon in a general way. It is used to lead into a discussion of the construction and theories of color perception of the eye. The reason for thus taking the eye away from the other optical instruments, and color away from considerations of wave-length, is apparently to make the early chapters more of general, diffuse discussion and therefore superficially easier. The gains seem to the reviewer to be somewhat doubtful.

The debate on "ether" vs. "space-properties" is handled reasonably, but it would seem as if the student's energy and time might be more profitably employed, since the discussion is, after all, so barren. The use of two names for the same thing only invites trouble for the beginner. The introduction of the electro-magnetic idea, early in this discussion, with frequent reference to the reaction between charges and such waves, would prepare the student for what of explanation there is in the modern view.

The subjects of formation of images and of optical instruments, generally, are treated almost entirely by the use of wave-fronts instead of rays. This serves the very desirable purpose of keeping before the reader the fact that the subject being discussed is wave-motion; but the geometry is not so simple, and in the complex cases the figures, at least, revert to the ray construction. Like many other complementary treatments, each has superior elements and any reasonably full discussion has to make use of both. There is also no doubt that here the emphasis on the wave treatment is the better introduction. The treatment in this whole section is not so complete as in many of the introductory textbooks. No suggestion is given of any more complete treatment: for instance, nodal points or nodal planes are never mentioned; the wave-front argument neglects the optical center, which is not mentioned; the geometrical construction for the formation of images is used in some of the figures, but the method receives no consideration; no emphasis is placed on the exceedingly approximate character of such formulae as are given—the whole tone of the treatment leaves an impression of finality, so that not even the brightest student would guess that whole volumes are available in which the present treatment is relegated to the scrap pile. One bright spot is the carrying through of a numerical calculation for an achromatic lens.

The discussion of simple diffraction phenomena leading up to resolving power, of simple cases of interference, of the formulation and application of the equations of simple harmonic motion, and of polarization phenomena is much more adequate than the earlier chapters and is clear and direct. It errs, if at all, on the side of being too elementary.

It is pleasant to find the electro-magnetic theory discussed here in place of the elastic-solid theory, which one still finds in some semi-elementary books. The discussion is sufficient to connect the electrical properties and light properties. X-rays must be classified now as electro-magnetic waves, and, as such, their nearest relative is what has been classed loosely as light. Optical books should discuss their properties. The discussion here is adequate for the purpose as it revolves about the relation of X-rays to light.

In a general way, there is here but little more intellectual food than in many of the fuller texts on general physics. There are reasons for not desiring to use a general text for a special course on one of its subdivisions. All these general texts contain far more than the sophomore student is able to assimilate. All that can be done in a second course is to present the same material in a little different dress, with some slight extensions. Mr. Reese's book serves this purpose excellently.

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